EXPTIME-complete Decision Problems for Modal and Mixed Specifications

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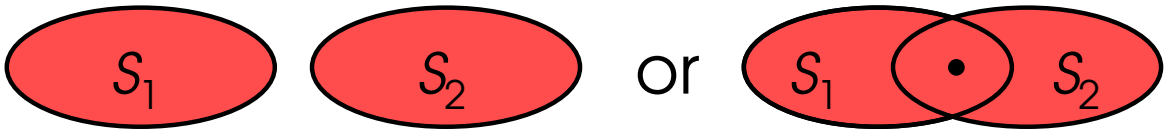
We Ask Complexity Questions For

CI Common Implementation



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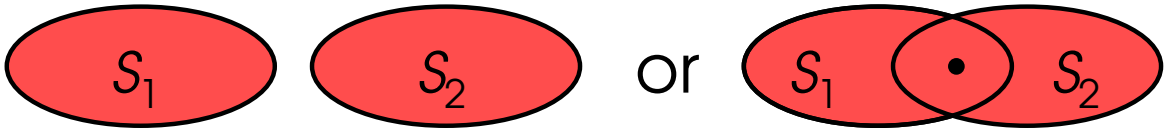

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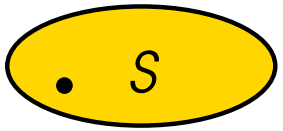
C Consistency

$S = \varnothing$ or



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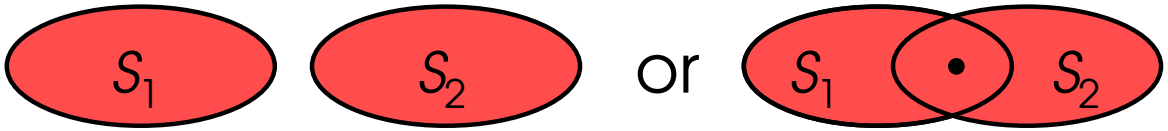

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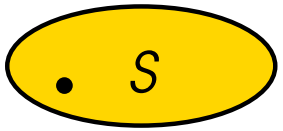

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TR Thorough Refinement

?

Agenda

- Modal and Mixed **Specifications** in a Nutshell
- The Problems and Our **Claims**
- Some **Proof** Sketches
- Open Issues & **Summary**

Part I

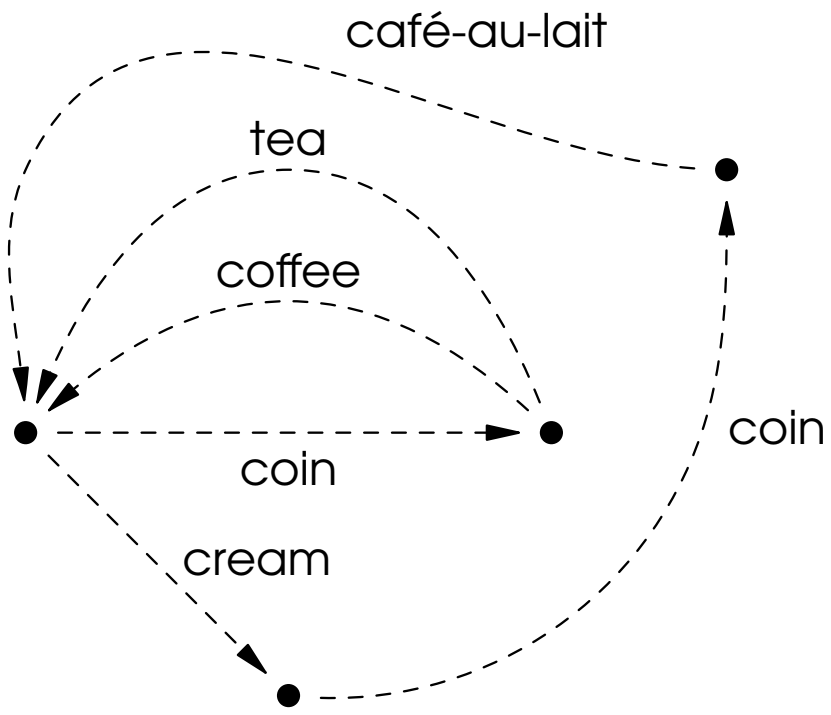
Modal & Mixed Specifications in A Nutshell

Labeled Transition Systems

A Coffeemaker Example

Some traces of the coffeemaker:

- insert coin, get coffee
- insert coin, get tea
- press cream, insert coin, get café au lait

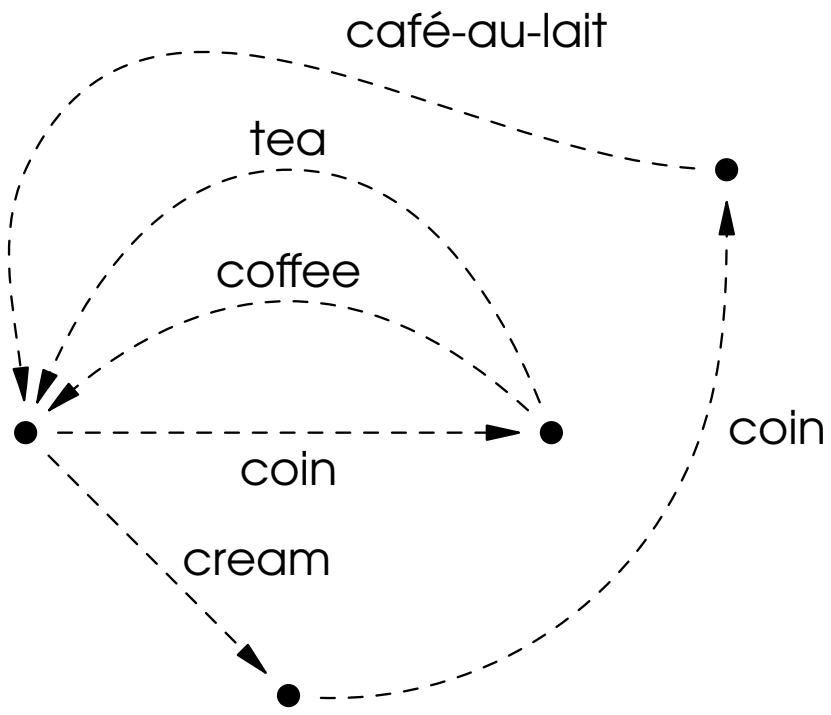


Labeled Transition Systems

A Coffeemaker Example

An LTS + simulation refinement

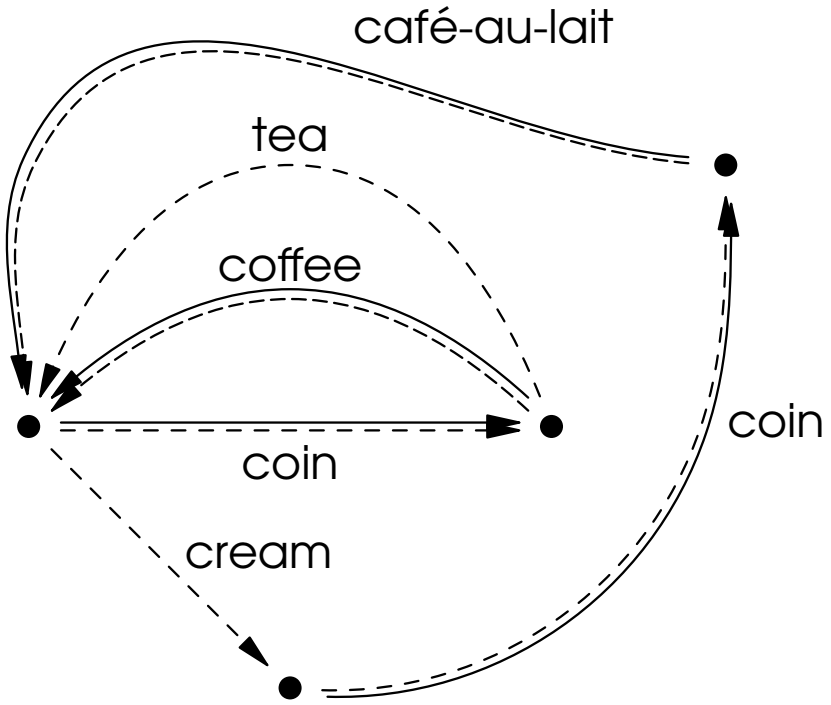
- Overapproximate possible behaviors in each state
- An empty LTS "•" is a perfect refinement.



Modal Specifications

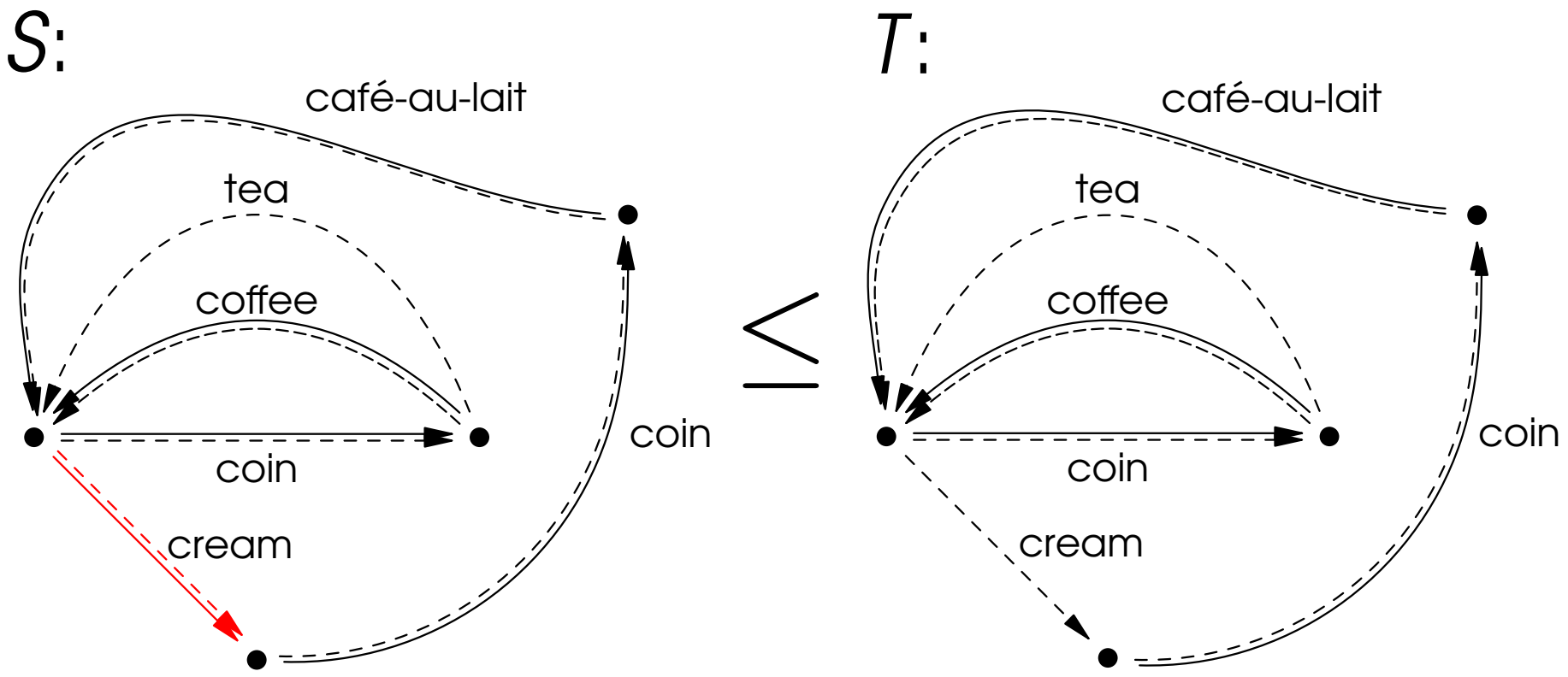
Larsen & Thomsen, LICS'88

- Under- and over-approximate behavior
- Each implementation **must** accept coins and produce coffee
- Cream or tea optional
- If cream offered then caffe-au-lait must be delivered



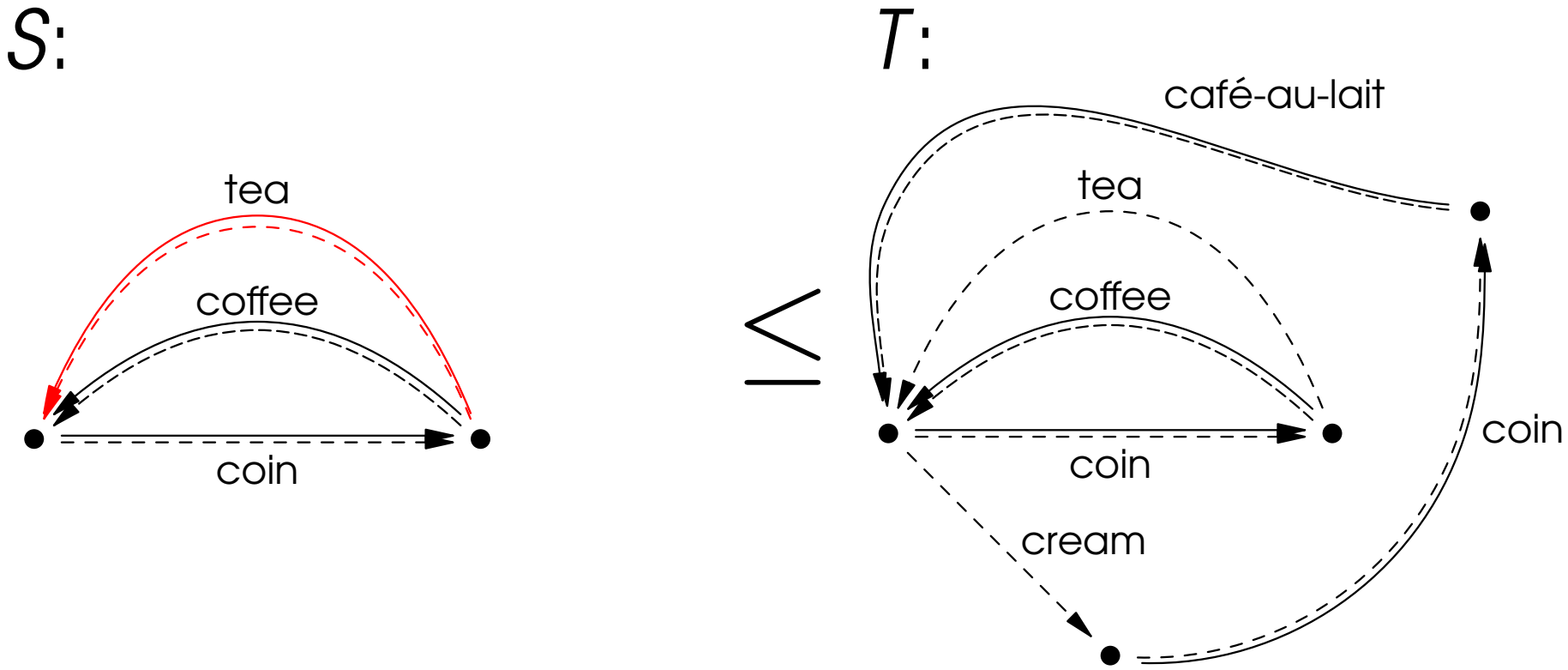
All required behavior (**must**) is allowed (**may**).

Refinement



May refines to must, may or nothing. Must refines to must.

Refinement



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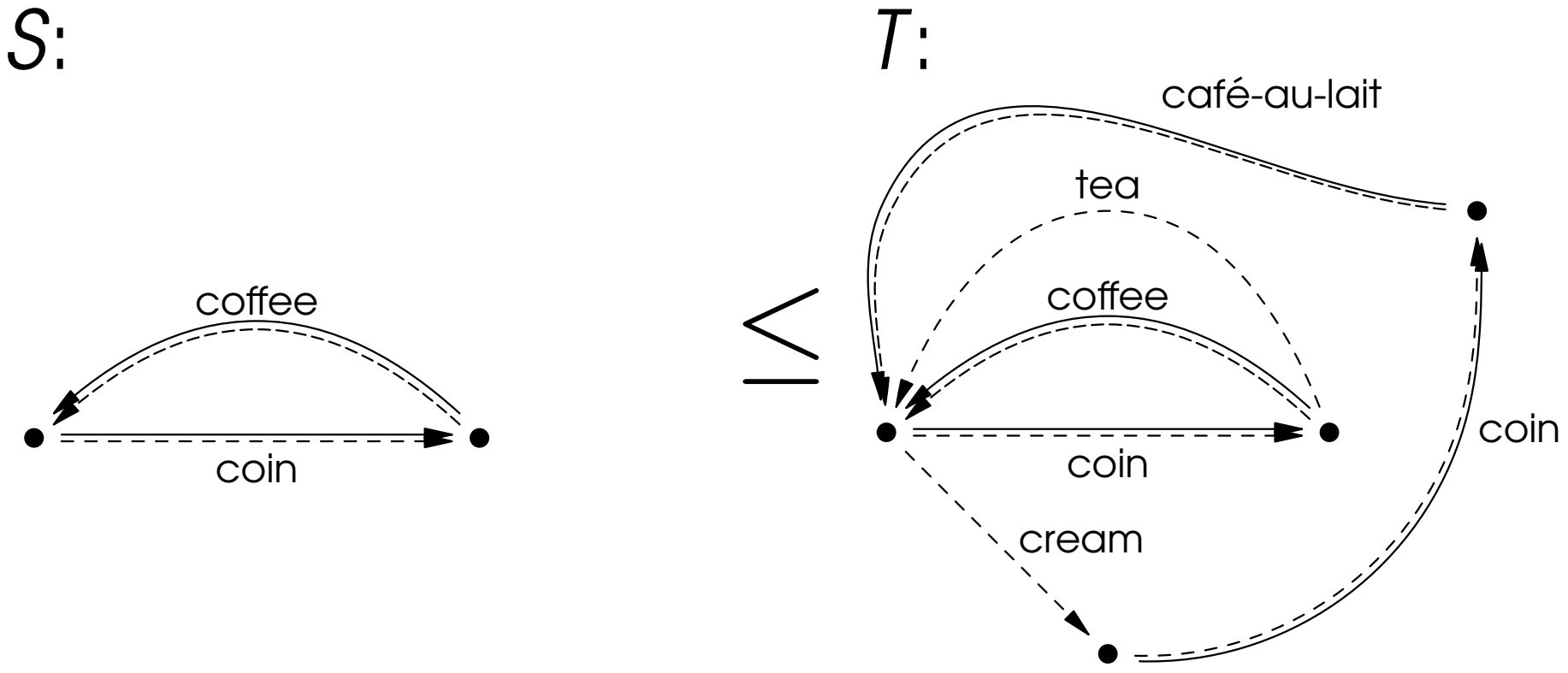
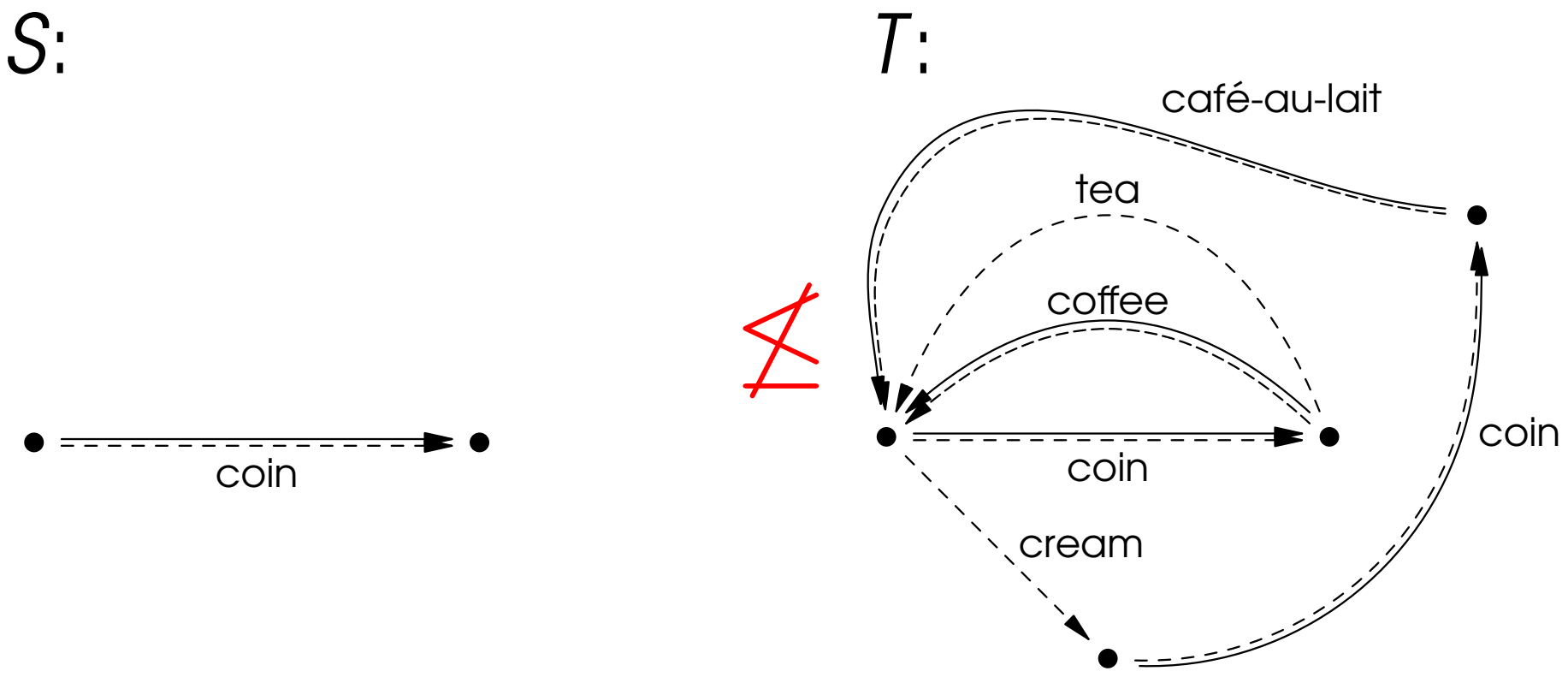
Refinement



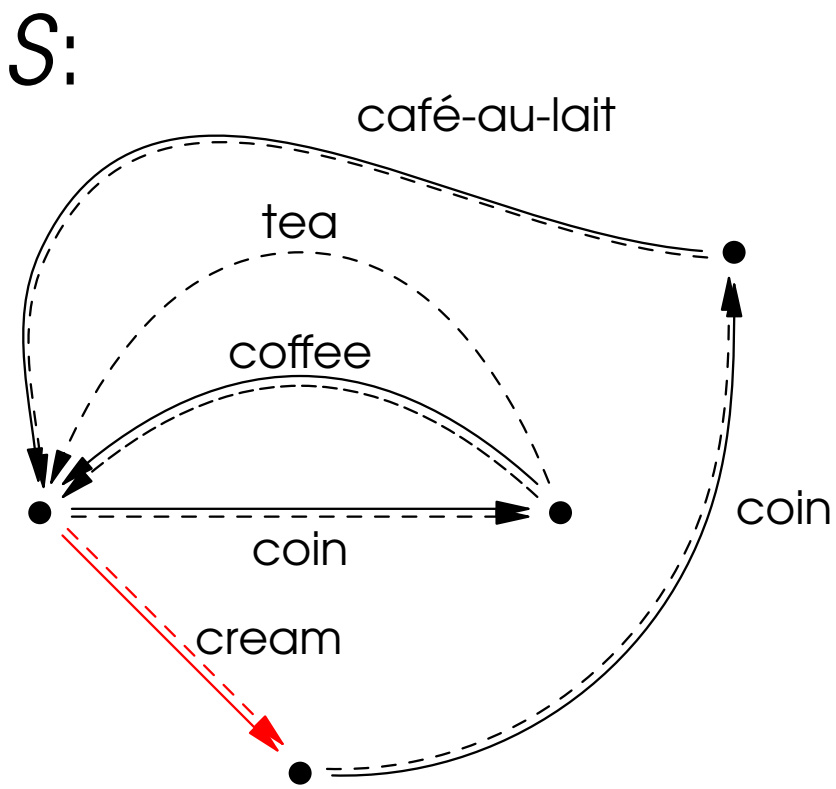
Infinitely many more refinements exist!!!

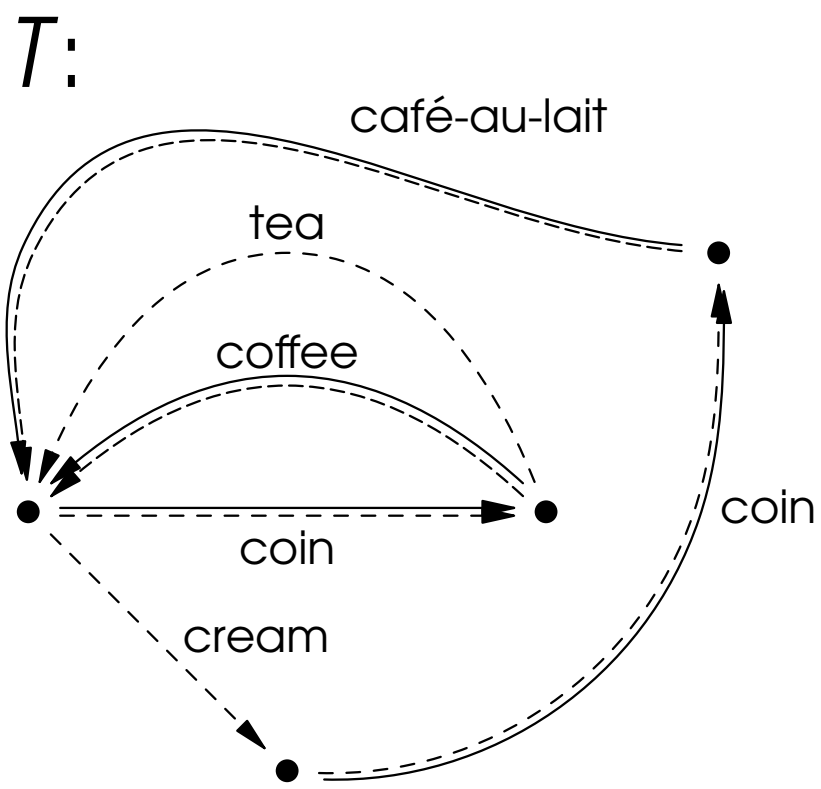
Refinement



But this is not a refinement!

Refinement



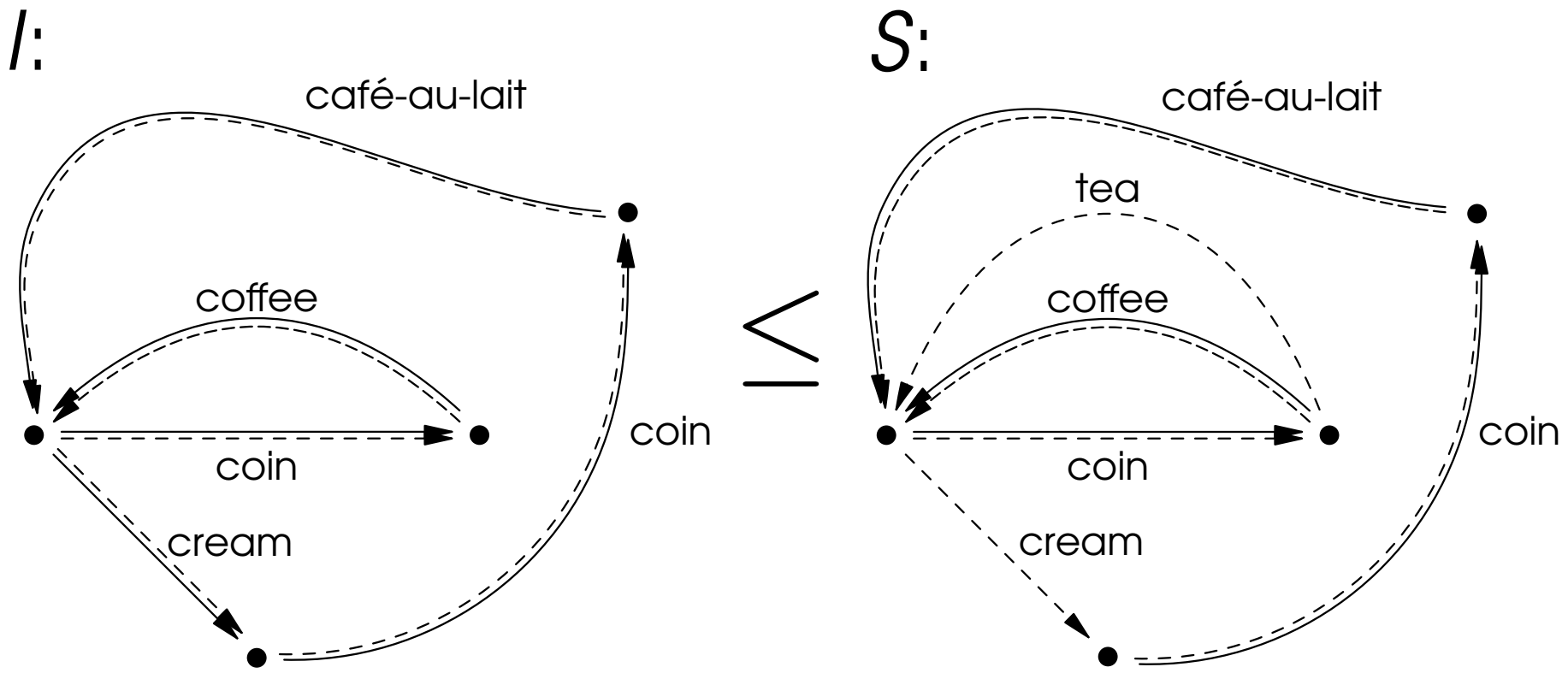


A relation $\leq$ is refinement iff for every $s \leq t$ it holds that

whenever $s \dashrightarrow^{a} s'$ then also $t \dashrightarrow^{a} t'$ for some t' and $s' \leq t'$

whenever $t \xrightarrow{a} t'$ then also $s \xrightarrow{a} s'$ for some s' and $s' \leq t'$

Implementations

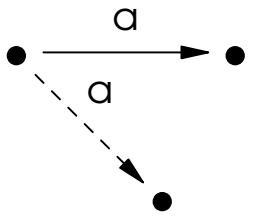


I is an implementation of S iff

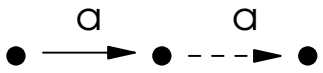
$$I \leq S \text{ and } \longrightarrow_I \; = \; \dashrightarrow_I$$

Mixed vs Modal Specifications

- **Modal** specifications: $\longrightarrow \subseteq \dashrightarrow$
  - $\rightarrow$ Always have implementations (consistent)
- **Mixed** specifications: possibly $\longrightarrow \not\subseteq \dashrightarrow$
  - $\rightarrow$ Larsen'89, Dams'96
- A **consistent** mixed specification:



- An **inconsistent** mixed specification:



Why Modal & Mixed Specifications ?

- **Semantic foundation** for specification & verification
- Same spec **combines** under- & over-approximations
  $\rightarrow$ existential and universal properties in static analysis
- Refinement is the **mid-way** between simulation (too weak) & bisimulation (too strong)
- See **recent survey** by the authors for more applications and more results
  $\rightarrow$ Bulleting of EATCS, June 2008

Part II

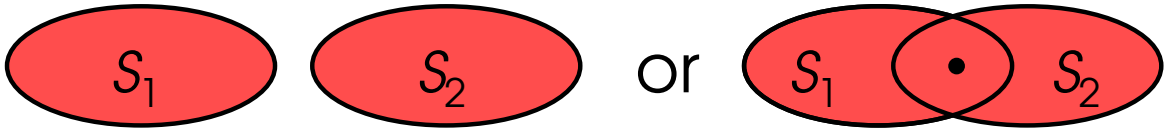
The Problems
&
Our Claims

Common Implementation

Problem CI

For modal (mixed) specifications S_1 and S_2 decide if

$$\exists \text{ implementation } I.\ I \leq S_1 \text{ and } I \leq S_2$$



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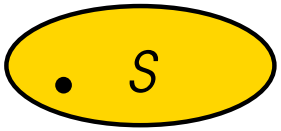
Claim: EXPTIME-complete

Consistency

Problem C

For a **mixed** specification S decide if

$$\exists \text{ implementation } I.\ I \leq S$$

$S = \varnothing$ or  **?**

Claim: EXPTIME-complete

Remark: this problem is trivial for **modal** specifications.

Thorough Refinement

Problem TR

For a **mixed** specifications S_1 and S_2 decide if

$$\forall \text{ implementations } I.\ I \leq S_1 \text{ implies } I \leq S_2$$

?

Claim: EXPTIME-complete

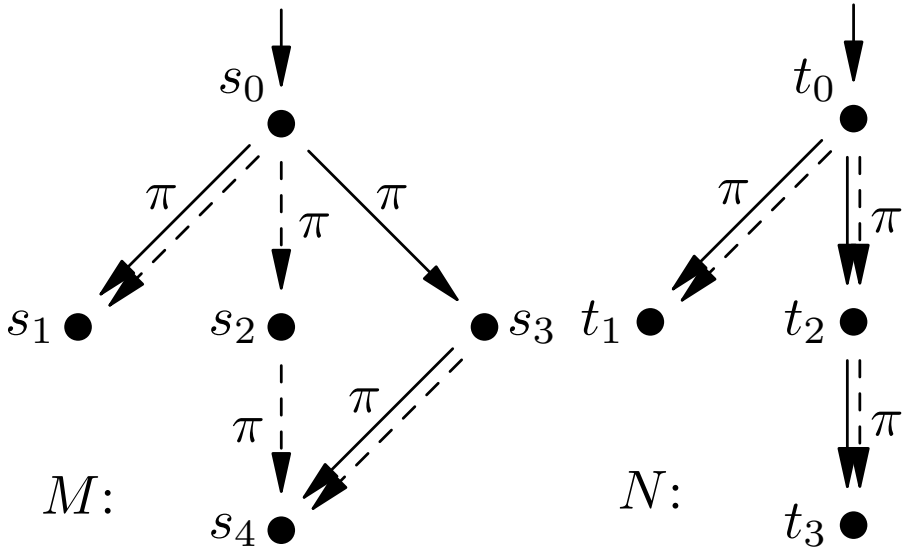
Remark: this problem is open for **modal** specifications.

Refinement vs Thorough Refinement

Note that refinement is in P, while TR is EXPTIME-complete.

So Refinement and TR **do not coincide**.

(Hüttel'88) proves this using a counterexample in this spirit:



Implementations sets of M and N are equal, but $M \not\leq N$.
Similar examples exist for properly modal specifications.

Part III

Proof Sketches

Bounds Before This Work

Antonik et al. FOSSACS'08

	Modal spec.	**Mixed spec.**
CI	PSPACE-hard, EXPTIME	PSPACE-hard, EXPTIME
C	trivial	PSPACE-hard, EXPTIME
TR	PSPACE-hard, EXPTIME	PSPACE-hard, EXPTIME

FOSSACS'08:

- Two complicated reductions showing the red !'s.
- A chain of reductions along the red arrows.

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New Bounds — The Proof Structure

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- Prove hardness of CI for modal specifications
- By the know sequence of reductions arrive at the remaining results
- So far failed to reduce TR in the modal case

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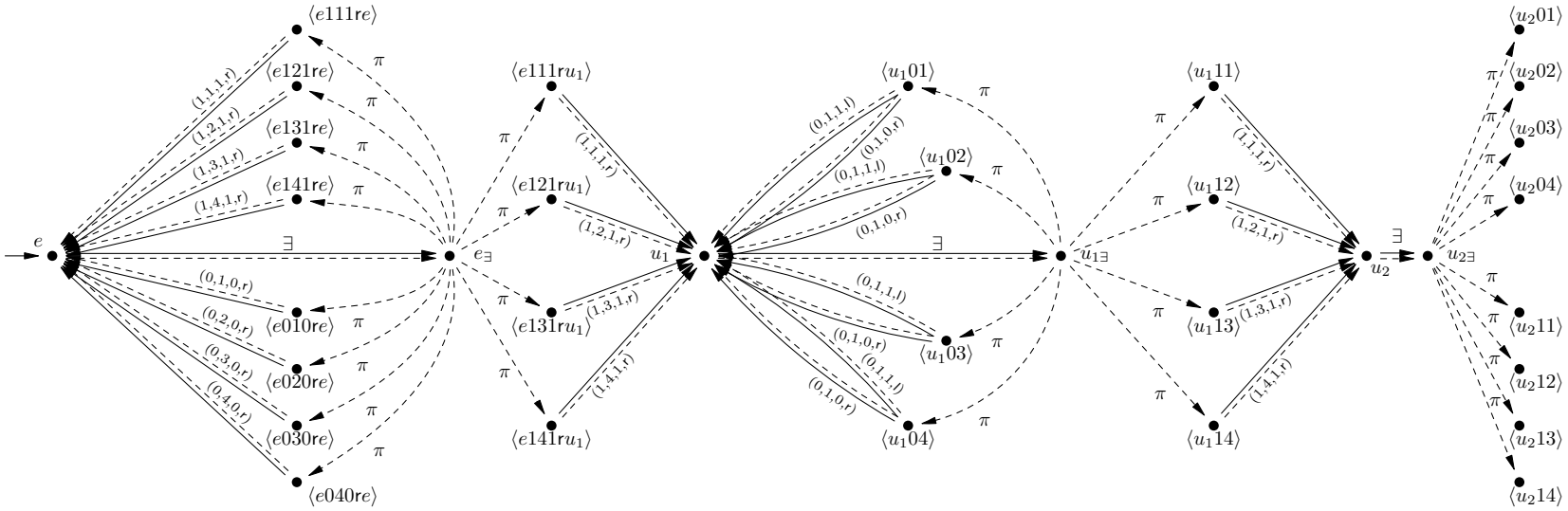
CI for Modal Specs is EXPTIME-complete

Most of the paper is devoted to EXPTIME-completeness of CI for Modal Specifications

The proof is by reduction from the acceptance problem for linearly bounded alternating Turing machines.

A teaser:



More in the paper.

Part IV

Closing

Summary

	Modal specifications	Mixed specifications
CI	**EXPTIME-complete**	**EXPTIME-complete**
C	trivial	**EXPTIME-complete**
TR	PSPACE-hard, EXPTIME	**EXPTIME-complete**

New results in **bold**.
The remaining gap in red.