Büchi Conditions for Generalized Energy Automata

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Motivation

Lower bound problems for energy automata, examples:

- Given finite automaton with integer weights on transitions: does there exist an infinite run in which the accumulated weight never drops below 0?
  - decidable in P
  - Bouyer-F.-Larsen-Markey-Srba:FORMATS'08
- Given timed automaton with integer weights on edges and integer rates in locations: decide the same problem
  - decidable for 1 clock; high complexity
  - by reduction to finite automata with special weight update functions on transitions
  - Bouyer-F.-Larsen-Matkey:HSCC'10
- Proof principle: if there's an infinite run, then there's a "lasso"

Goal: Generalize. What's the natural setting?

Energy Automata

Energy function:

- right-continuous autofunction f on $\{\bot\} \cup \mathbb{R}_{\geq 0} \cup \{\infty\}$
- $\bot$ means "undefined"
- $f(\bot) = \bot$, $f(\infty) = \infty$
- total order: $\bot < x < \infty$
- for $x_1 \leq x_2$: $f(x_2) - f(x_1) \geq x_2 - x_1$
  - "derivative $f' \geq 1$"
- so $f(x) = \bot$ implies $f(x') = \bot$ for all $x' \leq x$: f is defined on a left-closed interval

Energy automaton:

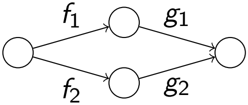
- finite automaton with transitions labeled with energy functions
- transitions "transform energy" input $\mapsto$ output
- $f(x) = \bot$ for an f-labeled transition: transition is not enabled for input x

Energy Function Semiring

Interest: Büchi acceptance

- Given a set F of accept states and $x_0 \in \mathbb{R}_{\geq 0}$: does there exist a run with initial energy x_0 which visits F infinitely often?

Operations on energy functions: max and $\circ$

- 

becomes $\max(g_1 \circ f_1, g_2 \circ f_2)$

The set $\mathcal{E}$ of energy functions with operations max and $\circ$ is a semiring, with $\mathbb{0} = \lambda x.\bot$, $\mathbb{1} = \lambda x.x$

- without "$f' \geq 1$" condition, only "near-semiring"
- idempotent, positively ordered, complete

$*$ and ω

Star: $f^* = \sup_{i \geq 0} f^i$

- for loops which can be taken an arbitrary number of times
- $f^*(x) = \begin{cases} x & \text{if } f(x) \leq x \\ \infty & \text{if } f(x) > x \end{cases}$

Omega: " $f^\omega = \lim_{i \to \infty} f^i$ "

- for loops which are taken infinitely often
- $f^\omega(x) = \begin{cases} \bot & \text{if } f(x) < x \\ x & \text{if } f(x) = x \\ \infty & \text{if } f(x) > x \end{cases}$

Reachability

Special case of Büchi acceptance: reachability

- Given automaton (S, M) ($M : S \times S \to \mathcal{E}$ is the transition matrix) and $s_0, s_f \in S$: is s_f reachable from s_0 with initial energy x_0?
- Theorem: Let $M^* : S \times S \to \mathcal{E}$ be the closure of M. Then s_f is reachable from s_0 with initial energy x_0 iff $M^*(s_0, s_f)(x_0) \neq \bot$.
- So (not surprisingly) M^* captures precisely reachability, in a nice static way: compute M^* once, and solve all reachability problems.

Büchi Acceptance

Theorem: Let $(S, M : S \times S \to \mathcal{E})$, $F \subseteq S$, $s_0 \in S$, $x_0 \in \mathbb{R}_{\geq 0}$.
There is a run from (s_0, x_0) which visits F infinitely often iff

- there is a run from (s_0, x_0) to some (s, x) with $s \in F$,
- and a loop from (s, x) to (s, x') with $x' \geq x$

Question: Is there also a nice static way to express this property?

(And relaxing any of the conditions on energy functions quickly leads to undecidability.)