

# History-Preserving Bisimilarity for Higher-Dimensional Automata via Open Maps

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(*cf.* MFPS)

- **History-preserving bisimilarity** is, “morally”, a relation on *paths*
- But we can show that for **higher-dimensional automata**, it is equivalent to a relation on *states* and (higher-dimensional) *transitions*
- This adds weight to the claim that HDA are a natural and useful (and beautiful!) formalism for concurrency

# Higher-Dimensional Automata

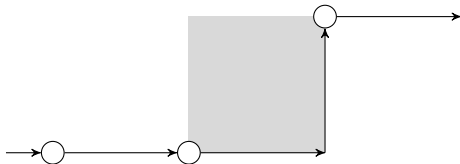
- Formalism for **concurrency**
- Generalizes Petri nets and most others
- Invented by V. Pratt and R. van Glabbeek
- Like automata, but with **higher-dimensional objects** which signify **independence / concurrency**
- States, transitions, squares, cubes, etc.

# Simple Bisimilarity

- Def.: HDA  $X, Y$  **om-bisimilar** if exists **sub-HDA**  $R \subseteq X \times Y$  s.t. for all reachable  $x \in X, y \in Y$  with  $(x, y) \in R$ :
  - for all  $x = \delta_k^0 x'$ , there is  $y = \delta_k^0 y'$  with  $(x', y') \in R$
  - for all  $y = \delta_k^0 y'$ , there is  $x = \delta_k^0 x'$  with  $(x', y') \in R$
- Easy generalization of standard bisimilarity for transition systems
- Comes from a natural notion of **open maps**
- But how does it relate to standard notions of concurrent bisimilarity?

# History-Preserving Bisimilarity

- Hp-bisimilarity: relation on **computations** which respects **extensions** and **independence**
- For HDA:
  - computations = **cube paths**



- independence = **homotopy** (simple combinatorial notion)

# Main Result

- Def.: HDA  $X, Y$  **hp-bisimilar** if exists relation  $R$  between cube paths in  $X$  and cube paths in  $Y$  s.t. for all  $(\rho, \sigma) \in R$ :
  - for all  $\rho \rightsquigarrow \rho'$ , there is  $\sigma \rightsquigarrow \sigma'$  with  $(\rho', \sigma') \in R$ ,
  - for all  $\sigma \rightsquigarrow \sigma'$ , there is  $\rho \rightsquigarrow \rho'$  with  $(\rho', \sigma') \in R$ ,
  - for all  $\rho \sim \rho'$ , there is  $\sigma \sim \sigma'$  with  $(\rho', \sigma') \in R$ ,
  - for all  $\sigma \sim \sigma'$ , there is  $\rho \sim \rho'$  with  $(\rho', \sigma') \in R$ ,
- **Theorem:** HDA are hp-bisimilar iff they are om-bisimilar.
- Proof via **unfoldings** of HDA into higher-dimensional trees (**universal covering**)

- **Coalgebraic** characterization?
- Relation to Staton-Winskel's (LICS 2010) unfolding of HDA into presheaves over symmetric **event structures**?
- **Hereditary** hp-bisimilarity?