Higher-Dimensional Timed Automata

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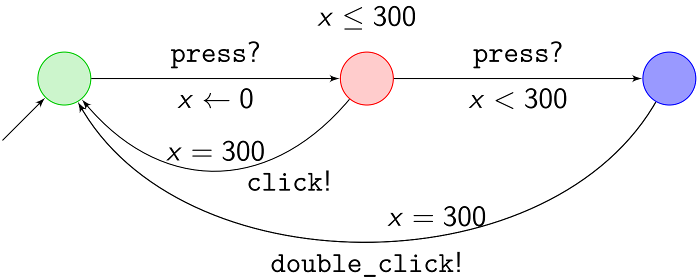
Recall Timed Automata



Recall Timed Automata

Definition

The set $\Phi(C)$ of clock constraints ϕ over a finite set C is defined by the grammar

$$\phi ::= x \bowtie k \mid \phi_1 \wedge \phi_2 \qquad (x, y \in C, k \in \mathbb{Z}, \bowtie \in \{\leq, <, \geq, >\}).$$

Definition

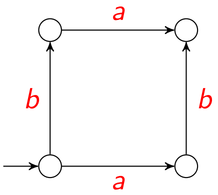
A timed automaton is a tuple $(L, \ell_0, C, \Sigma, I, E)$ consisting of a finite set L of locations, an initial location $\ell_0 \in L$, a finite set C of clocks, a finite set Σ of actions, a location invariants mapping $I : L \to \Phi(C)$, and a set $E \subseteq L \times \Phi(C) \times \Sigma \times 2^C \times L$ of edges.

Recall Timed Automata

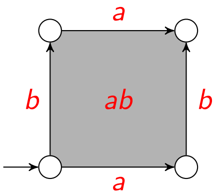
- Useful for modeling synchronous real-time systems
- Reachability, emptiness, LTL model checking PSPACE-complete
- Universality undecidable
- Fast on-the-fly algorithms, using zones, for reachability, liveness, and Timed CTL model checking
- UppAal
- Extensions to weighted timed automata, real-time games, etc.

Recall Higher-Dimensional Automata

$a|b$



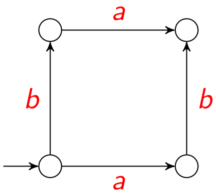
$a.b + b.a$



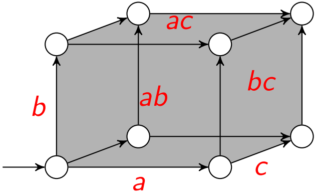
a and b are independent

Recall Higher-Dimensional Automata

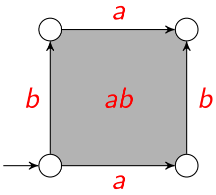
$a|b$

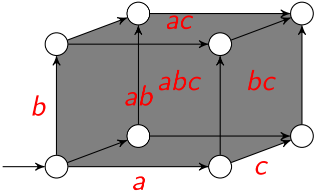


$a|b|c$



$a|b + a|c + b|c$



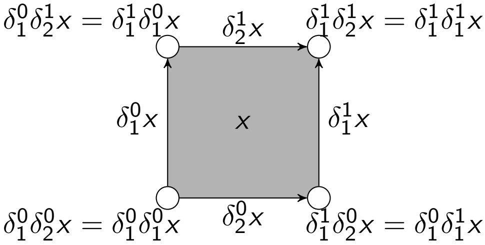


$\{a, b, c\}$ independent

Recall Higher-Dimensional Automata

A precubical set:

- a graded set $X = \{X_n\}_{n \in \mathbb{N}}$
- in each dimension n, $2n$ face maps $\delta_k^0, \delta_k^1 : X_n \to X_{n-1}$ $(k = 1, \dots, n)$
- the precubical identity: $\delta_k^\nu \delta_\ell^\mu = \delta_{\ell-1}^\mu \delta_k^\nu$ for all $k < \ell$



A higher-dimensional automaton: a finite precubical set with initial state and accepting states

Recall Higher-Dimensional Automata

HDA as a model for concurrency:

- points $x \in X_0$: states
- edges $a \in X_1$: transitions (labeled with events)
- n-squares $\alpha \in X_n$ ($n \geq 2$): independency relations (concurrently executing events)

van Glabbeek (TCS 2006): Up to history-preserving bisimilarity, HDA generalize "the main models of concurrency proposed in the literature"

- (for example Petri nets)

The Marriage between Real Time and Concurrency

- In real-time formalisms, everything is synchronous
  - timed automata, timed Petri nets, hybrid automata, etc.
- and concurrency is interleaving

- In formalisms for (non-interleaving) concurrency, no real time
  - same for distributed computing theory
  - (Petri nets have a concurrent semantics; timed Petri nets don't)

- Our goal: formalisms for real-time concurrent systems
- Application: for example distributed cyber-physical systems

- Here: the marriage between timed and higher-dimensional automata

Actions Take Time?

- Cardelli 1982 (ICALP): Actions take time.
  - 'We read $p \xrightarrow[t]{a} q$ as "p moves to q performing a for an interval t"'
- since Alur-Dill 1990 (even before?): Actions are immediate.
  - $(l, v) \overset{d}{\rightsquigarrow} (l, v+d) \overset{s}{\rightsquigarrow} (l', v+d)$
- Kim G. Larsen (many personal discussions): Actions are immediate because of tradition. ("This is how we know how to do.")
- Chatain-Jard 2013: In the concurrent semantics for time Petri nets, time has to (locally) be allowed to run backwards??
- U.F. 2018: In real-time concurrency, actions cannot be immediate.
  - and it appears that the "technical reasons" argument is quite weak!

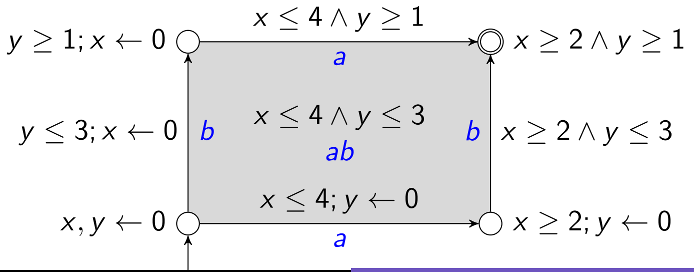
Higher-Dimensional Timed Automata

Definition

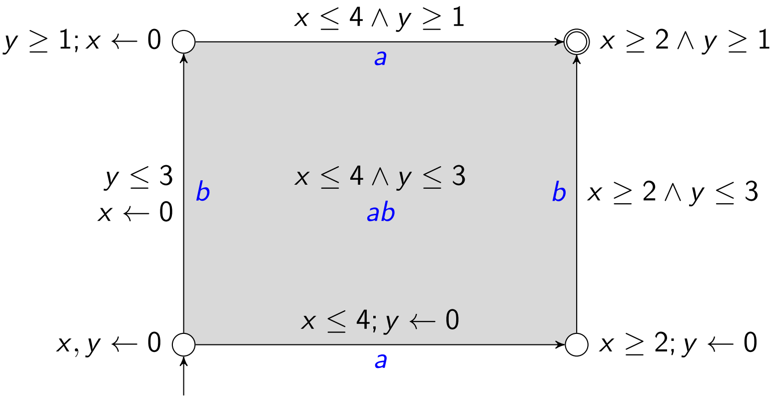
A HDTA is a structure $(L, l^0, L^f, \lambda, C, \mathrm{inv}, \mathrm{exit})$, where (L, l^0, L^f, λ) is a finite HDA, C is a finite set of clocks, and $\mathrm{inv} : L \to \Phi(C)$, $\mathrm{exit} : L \to 2^C$ give invariant and exit conditions for each n-cube.

Intuition:

- $\mathrm{inv}(l)$: conditions on the clock values while delaying in l
- $\mathrm{exit}(l)$: clocks to be reset to 0 when leaving l.

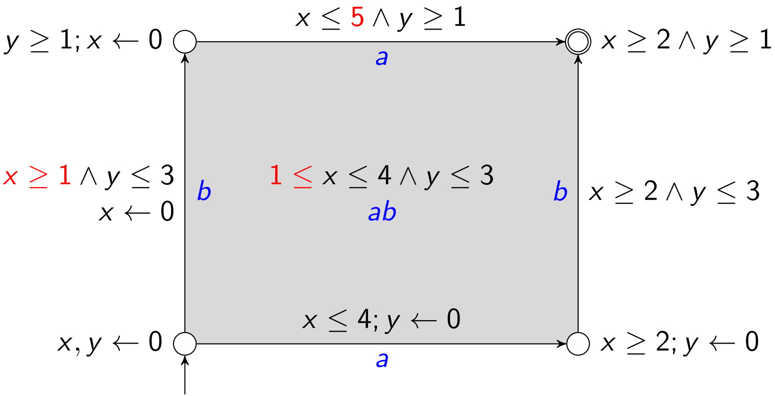


Examples



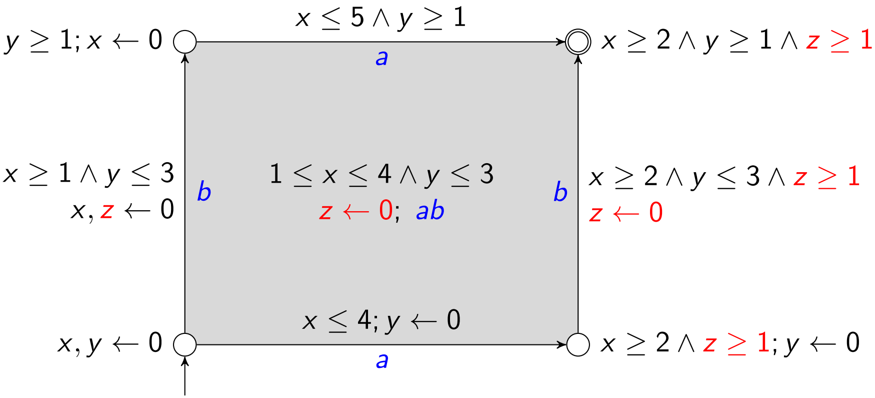
- a takes $[2, 4]$ time units, b takes $[1, 3]$ time units

Examples



- a takes $[2, 4]$ time units, b takes $[1, 3]$ time units
- unless b is done before a
- b can only start 1 time unit after a

Examples

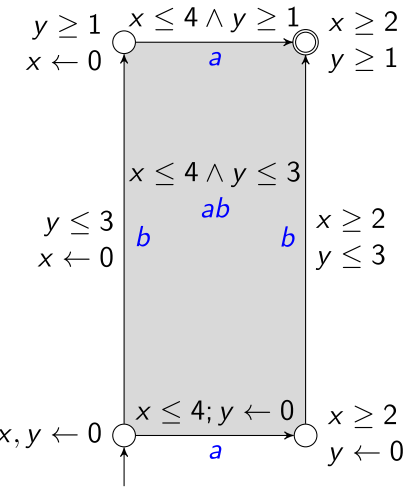


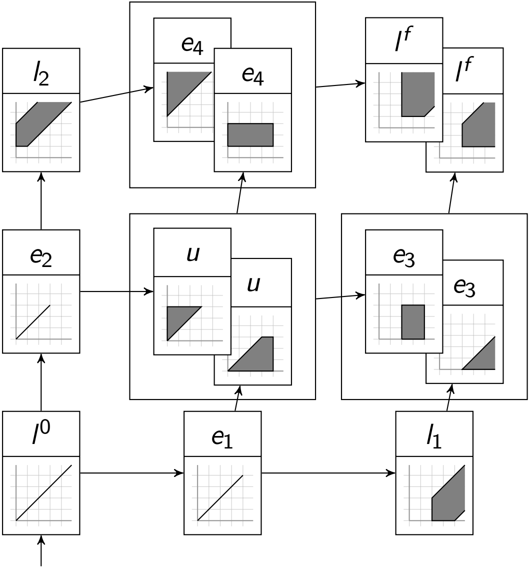
- a takes $[2, 4]$ time units, b takes $[1, 3]$ time units
- b can only start 1 time unit after a
- b has to finish 1 time unit before a

Good News

- Reachability for HDTA is PSPACE-complete
- and can be checked using zone-based algorithms
- (Everything works like for timed automata)
- Universality probably still undecidable

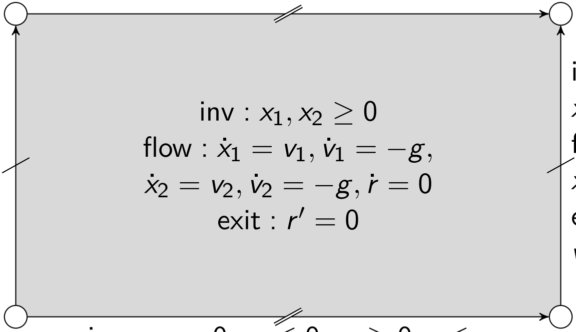
Zone-Based Reachability





Higher-Dimensional Hybrid Automata

Two independently bouncing balls (with temporal regularization):

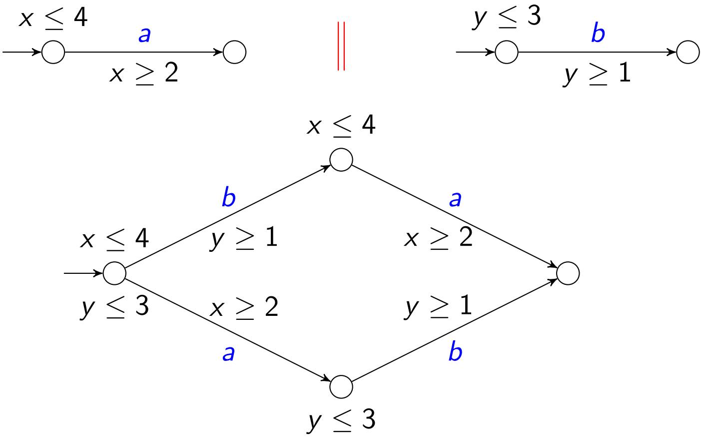


Conclusion

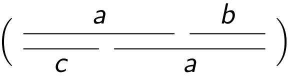
- Higher-dimensional timed automata: a nice formalism for real-time concurrency?
- Also, higher-dimensional hybrid automata
- For HDTA verification, zones
- Tensor product for parallel composition
- “Partial-order reduction built in”
- Actions should take time!?

Parallel Composition of Timed Automata

a takes between 2 and 4 time units ‖ *b* takes between 1 and 3 time units

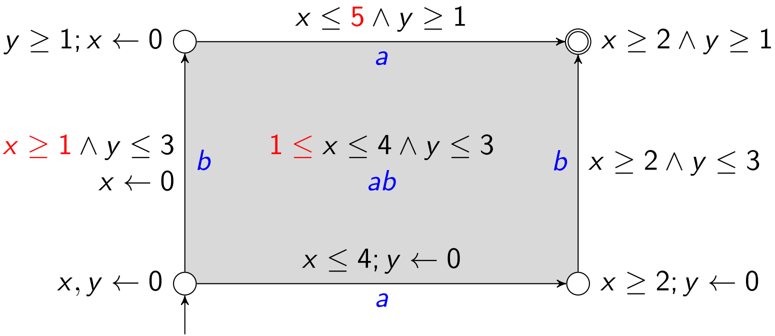


Languages of HDTA

- Timed automata generate timed words (w, t):
  - $w = w_1 \dots w_n \in \Sigma^*$
  - $t = (t_1, \dots, t_n) \in \mathbb{R}^n_{\geq 0}$ increasing sequence of time stamps
  - example: $\begin{pmatrix} a & c & a & a \\ .7 & 1.1 & 1.1 & 1.7 \end{pmatrix}$
- Higher-dimensional automata generate labeled interval orders (I, ℓ):
  - $I = \{[l_i, r_i]\} \subseteq \mathbb{N} \times \mathbb{N}$ finite set of intervals ($l_i \leq r_i$)
  - $\ell : I \to \Sigma$
  - example: 

- Proposal: HDTA generate timed interval orders (I, ℓ):
  - $I = \{\!\{[l_i, r_i]\}\!\} \subseteq \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}$ finite multiset of real intervals ($l_i \leq r_i$)
  - $\ell : I \to \Sigma$

Example



$$L(A) = \left\{ \{\!\{ [l_1, r_1]^a, [l_2, r_2]^b \}\!\} \;\middle|\; \begin{cases} 1 \le r_2 - l_2 \le 3 \\ 2 \le r_1 - l_1 \le \begin{cases} 4 & \text{if } r_1 < r_2 \\ 5 & \text{if } r_1 \ge r_2 \end{cases} \\ l_2 \ge l_1 + 1 \end{cases} \right\}$$