

Behavioral Specification Theories

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Motivation

models

specifications

Mod

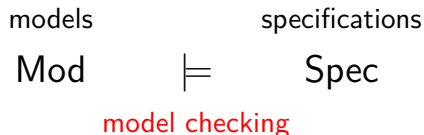
\models

Spec

model checking

Not so easy. . .

Motivation



Not so easy. . .

Incremental certification / **Compositional** verification

- bottom-up **and** top-down

Wish list:

- $\text{Mod} \models \text{Spec}_1 \ \& \ \text{Spec}_1 \leq \text{Spec}_2 \implies \text{Mod} \models \text{Spec}_2$
- $\text{Mod} \models \text{Spec}_1 \ \& \ \text{Mod} \models \text{Spec}_2 \implies \text{Mod} \models \text{Spec}_1 \wedge \text{Spec}_2$
- $\text{Mod}_1 \models \text{Spec}_1 \ \& \ \text{Mod}_2 \models \text{Spec}_2 \implies \text{Mod}_1 \parallel \text{Mod}_2 \models \text{Spec}_1 \parallel \text{Spec}_2$
- $\text{Mod}_1 \models \text{Spec}_1 \ \& \ \text{Mod}_2 \models \text{Spec} / \text{Spec}_1 \implies \text{Mod}_1 \parallel \text{Mod}_2 \models \text{Spec}$

Compositional Verification

- $\text{Mod} \models \text{Spec}_1 \ \& \ \text{Spec}_1 \leq \text{Spec}_2 \implies \text{Mod} \models \text{Spec}_2$
 - **incrementality**
- $\text{Mod} \models \text{Spec}_1 \ \& \ \text{Mod} \models \text{Spec}_2 \implies \text{Mod} \models \text{Spec}_1 \wedge \text{Spec}_2$
 - **conjunction**
- $\text{Mod}_1 \models \text{Spec}_1 \ \& \ \text{Mod}_2 \models \text{Spec}_2 \implies \text{Mod}_1 \parallel \text{Mod}_2 \models \text{Spec}_1 \parallel \text{Spec}_2$
 - **compositionality**
- $\text{Mod}_1 \models \text{Spec}_1 \ \& \ \text{Mod}_2 \models \text{Spec}/\text{Spec}_1 \implies \text{Mod}_1 \parallel \text{Mod}_2 \models \text{Spec}$
 - **quotient**

Not so easy – but **easier than model checking?**

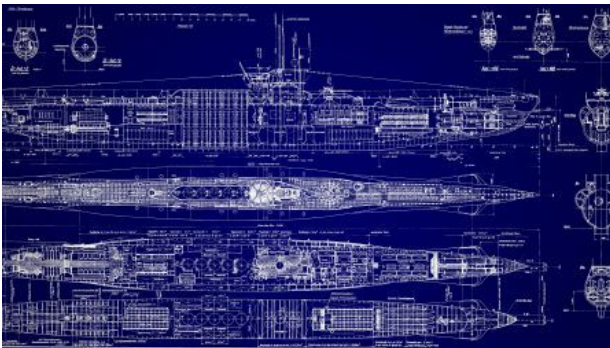
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 - **quotient**

Not so easy – but **easier than model checking?**

“Holy Grail”?

Application? Naval Group



- thousands of components; computing, physical, and mixed; from hundreds of subcontractors
- modern design needs formal(ish) verification
- what if between verification and implementation, a subcontractor decides to **improve a component??**

- 1 Motivation
- 2 **Acceptance Automata**
- 3 Specification Theories for Real Time, Probabilities, etc.
- 4 Conclusion

Acceptance Automata

Let Σ be a finite alphabet.

Definition

A (nondeterministic) **acceptance automaton** (AA) is a structure $\mathcal{A} = (S, S^0, \text{Tran})$, with $S \supseteq S^0$ finite sets of states and initial states and $\text{Tran} : S \rightarrow 2^{2^{\Sigma \times S}}$ an assignment of *transition constraints*.

- standard labeled transition system (**LTS**): $\text{Tran} : S \rightarrow 2^{\Sigma \times S}$ (**coalgebraic** view)
- (for AA:) $\text{Tran}(s) = \{M_1, M_2, \dots\}$: **provide M_1 or M_2 or \dots**
- a **disjunctive** choice of **conjunctive** constraints
- [J.-B. Raclet 2008](#) (but deterministic)
- note multiple initial states

Acknowledgement

- This is joint work with Nikola Beneš, Benoît Delahaye, Jan Křetínský, Axel Legay, and Louis-Marie Traonouez
- based on papers at CONCUR 2013, FACS 2014, ICTAC 2014, and SOFSEM 2017
- subsequently in Soft Computing 22(4):2018, Information & Computation (to appear), and Logical & Algebraic Methods in Programming 110:2020

Refinement

Definition

Let $\mathcal{A}_1 = (S_1, S_1^0, \text{Tran}_1)$ and $\mathcal{A}_2 = (S_2, S_2^0, \text{Tran}_2)$ be AA.

A relation $R \subseteq S_1 \times S_2$ is a **modal refinement** if:

$$\textcircled{1} \quad \forall s_1^0 \in S_1^0 : \exists s_2^0 \in S_2^0 : (s_1^0, s_2^0) \in R \quad \text{(init)}$$

$$\textcircled{2} \quad \forall (s_1, s_2) \in R : \forall M_1 \in \text{Tran}_1(s_1) : \exists M_2 \in \text{Tran}_2(s_2) : \quad \text{(tran)}$$

$$\textcircled{1} \quad \forall (a, t_1) \in M_1 : \exists (a, t_2) \in M_2 : (t_1, t_2) \in R$$

$$\textcircled{2} \quad \forall (a, t_2) \in M_2 : \exists (a, t_1) \in M_1 : (t_1, t_2) \in R$$

Write $\mathcal{A}_1 \leq \mathcal{A}_2$ if there exists such a modal refinement.

- for any **constraint choice** M_1 there is a **bisimilar** choice M_2
- \mathcal{A}_1 has **fewer choices** than \mathcal{A}_2
- no more choices $\hat{=}$ only one $M \in \text{Tran}(s) \hat{=}$ **LTS**
- formally: an **embedding** $\chi : \text{LTS} \hookrightarrow \text{AA}$
such that $\chi(\mathcal{L}_1) \leq \chi(\mathcal{L}_2)$ iff \mathcal{L}_1 and \mathcal{L}_2 are **bisimilar**

A Step Back

Let **Mod** be a set of models with an equivalence \sim .

Definition

A (behavioral) **specification theory** for (Mod, \sim) consists of

- a set **Spec**,
- a preorder $\leq \subseteq \text{Spec} \times \text{Spec}$, and
- a mapping $\chi : \text{Mod} \rightarrow \text{Spec}$,

such that $\forall \mathcal{M}_1, \mathcal{M}_2 \in \text{Mod} : \mathcal{M}_1 \sim \mathcal{M}_2 \iff \chi(\mathcal{M}_1) \leq \chi(\mathcal{M}_2)$.

- write $\mathcal{M} \models \mathcal{S}$ for $\chi(\mathcal{M}) \leq \mathcal{S}$
- $\chi(\mathcal{M})$: **characteristic formula** for \mathcal{M} : $\mathcal{M}' \models \chi(\mathcal{M}) \iff \mathcal{M}' \sim \mathcal{M}$
- **incrementality**: $\mathcal{M} \models \mathcal{S}_1 \ \& \ \mathcal{S}_1 \leq \mathcal{S}_2 \implies \mathcal{M} \models \mathcal{S}_2$
- acceptance automata $\hat{=}$ disjunctive modal transition systems $\hat{=}$
Hennessy-Milner logic with maximal fixed points
- **safety properties**

Logical Operations

Let $\mathcal{A}_1 = (S_1, S_1^0, \text{Tran}_1)$ and $\mathcal{A}_2 = (S_2, S_2^0, \text{Tran}_2)$ be AA.

Disjunction: $\mathcal{A}_1 \vee \mathcal{A}_2 = (S_1 \dot{\cup} S_2, S_1^0 \dot{\cup} S_2^0, \text{Tran}_1 \dot{\cup} \text{Tran}_2)$

Conjunction: define $\pi_i : 2^{\Sigma \times S_1 \times S_2} \rightarrow 2^{\Sigma \times S_i}$ by

$$\pi_1(M) = \{(a, s_1) \mid \exists s_2 \in S_2 : (a, s_1, s_2) \in M\}$$

$$\pi_2(M) = \{(a, s_2) \mid \exists s_1 \in S_1 : (a, s_1, s_2) \in M\}$$

Let $\mathcal{A}_1 \wedge \mathcal{A}_2 = (S_1 \times S_2, S_1^0 \times S_2^0, \text{Tran})$ with

$$\text{Tran}((s_1, s_2)) = \{M \subseteq \Sigma \times S_1 \times S_2 \mid \\ \pi_1(M) \in \text{Tran}_1(s_1), \pi_2(M) \in \text{Tran}_2(s_2)\}$$

Theorem (for all LTS \mathcal{L} and AA $\mathcal{A}_1, \mathcal{A}_2$)

$$\mathcal{L} \models \mathcal{A}_1 \vee \mathcal{A}_2 \iff \mathcal{L} \models \mathcal{A}_1 \text{ or } \mathcal{L} \models \mathcal{A}_2$$

$$\mathcal{L} \models \mathcal{A}_1 \wedge \mathcal{A}_2 \iff \mathcal{L} \models \mathcal{A}_1 \ \& \ \mathcal{L} \models \mathcal{A}_2$$

Another Step Back

Let **Mod** be a set of models with an equivalence \sim .

Definition (ad hoc)

A specification theory $(\text{Spec}, \leq, \chi)$ for (Mod, \sim) is **nice** if (Spec, \leq) forms a **bounded distributive lattice up to $\leq \cap \geq$** .

- \Rightarrow have **least upper bound** \vee and **greatest lower bound** \wedge
- \Rightarrow bottom specification **ff** ($\forall \mathcal{M} \in \text{Mod} : \mathcal{M} \not\models \mathbf{ff}$)
- \Rightarrow top specification **tt** ($\forall \mathcal{M} \in \text{Mod} : \mathcal{M} \models \mathbf{tt}$)
- \Rightarrow double **distributivity**
 - everything **up to modal equivalence** $\equiv = \leq \cap \geq$
 - holds for acceptance automata, disjunctive modal transition systems, and Hennessy-Milner logic with maximal fixed points

Structural Operations: Composition

Let $\mathcal{A}_1 = (S_1, S_1^0, \text{Tran}_1)$ and $\mathcal{A}_2 = (S_2, S_2^0, \text{Tran}_2)$ be AA.

For $M_1 \subseteq \Sigma \times S_1$ and $M_2 \subseteq \Sigma \times S_2$, define

$$M_1 \parallel M_2 = \{(a, (t_1, t_2)) \mid (a, t_1) \in M_1, (a, t_2) \in M_2\}$$

(assumes CSP synchronization, but can be generalized)

Let $\mathcal{A}_1 \parallel \mathcal{A}_2 = (S_1 \times S_2, S_1^0 \times S_2^0, \text{Tran})$ with

$$\text{Tran}((s_1, s_2)) = \{M_1 \parallel M_2 \mid M_1 \in \text{Tran}_1(s_1), M_2 \in \text{Tran}_2(s_2)\}$$

Theorem (independent implementability)

For all AA $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4$:

$$\mathcal{A}_1 \leq \mathcal{A}_3 \ \& \ \mathcal{A}_2 \leq \mathcal{A}_4 \implies \mathcal{A}_1 \parallel \mathcal{A}_2 \leq \mathcal{A}_3 \parallel \mathcal{A}_4$$

Structural Operations: Quotient

Let $\mathcal{A}_1 = (S_1, S_1^0, \text{Tran}_1)$ and $\mathcal{A}_2 = (S_2, S_2^0, \text{Tran}_2)$ be AA.

Define $\mathcal{A}_1 / \mathcal{A}_2 = (S, S^0, \text{Tran})$:

- $S = 2^{S_1 \times S_2}$
- write $S_2^0 = \{s_2^{0,1}, \dots, s_2^{0,p}\}$ and let
$$S^0 = \{ \{(s_1^{0,q}, s_2^{0,q}) \mid q \in \{1, \dots, p\}\} \mid \forall q : s_1^{0,q} \in S_1^0 \}$$
- $\text{Tran} =$

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Structural Operations: Quotient

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- $\text{Tran} = \dots$

Theorem

For all AA $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$:

$$\mathcal{A}_1 \parallel \mathcal{A}_2 \leq \mathcal{A}_3 \iff \mathcal{A}_2 \leq \mathcal{A}_3 / \mathcal{A}_1$$

- up to \equiv , $/$ is the **adjoint** (or **residual**) of \parallel

A Step Back, Again

Let **Mod** be a set of models with an equivalence \sim .

Definition

A **complete specification theory** for (Mod, \sim) is $(\text{Spec}, \leq, \parallel, \chi)$ such that $(\text{Spec}, \leq, \chi)$ is a specification theory for (Mod, \sim) and $(\text{Spec}, \leq, \parallel)$ forms a **bounded distribute commutative residuated lattice up to \equiv** .

- $\Rightarrow \parallel$ distributes over \vee and has a unit U , up to \equiv
- $\Rightarrow \parallel$ has a residual $/$, up to \equiv
 - a **compositional algebra** of specifications: for example,

$$(\mathcal{S}_1 \wedge \mathcal{S}_2) / \mathcal{S}_3 \equiv \mathcal{S}_1 / \mathcal{S}_3 \wedge \mathcal{S}_2 / \mathcal{S}_3$$

$$\mathcal{S}_1 \parallel (\mathcal{S}_2 / \mathcal{S}_1) \leq \mathcal{S}_2 \quad (\mathcal{S}_1 \parallel \mathcal{S}_2) / \mathcal{S}_1 \leq \mathcal{S}_2$$

$$\perp \parallel \mathcal{S} \equiv \perp \quad \mathcal{S} / U \equiv \mathcal{S} \quad U \leq \mathcal{S} / \mathcal{S} \quad U \equiv \perp / \perp$$

$$(\mathcal{S}_1 / \mathcal{S}_2) / \mathcal{S}_3 \equiv \mathcal{S}_1 / (\mathcal{S}_2 \parallel \mathcal{S}_3)$$

$$(U / \mathcal{S}_1) \parallel (U / \mathcal{S}_2) \leq U / (\mathcal{S}_1 \parallel \mathcal{S}_2)$$

A Step Back, Again

Let **Mod** be a set of models with an equivalence \sim .

Definition

A **complete specification theory** for (Mod, \sim) is $(\text{Spec}, \leq, \parallel, \chi)$ such that $(\text{Spec}, \leq, \chi)$ is a specification theory for (Mod, \sim) and $(\text{Spec}, \leq, \parallel)$ forms a **bounded distribute commutative residuated lattice up to \equiv** .

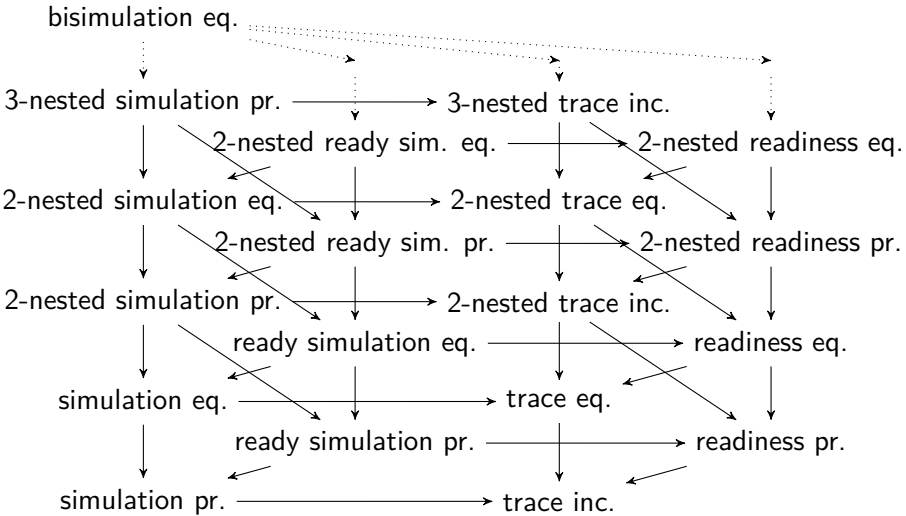
- $\Rightarrow \parallel$ distributes over \vee and has a unit \mathbb{U} , up to \equiv
- $\Rightarrow \parallel$ has a residual $/$, up to \equiv
 - a **compositional algebra** of specifications
 - relation to **linear logic** and **Girard quantales**

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Specification Theories for LTS

- (disjunctive) modal transition systems: [Larsen-Xinxin 1989-90]
- equivalence with acceptance automata and Hennessy-Milner logic with greatest fixed points: [Larsen-Boudol 1992], [Beneš-Delahaye-UF *et al.* 2013]
- modal transition systems **with data**: [Bauer-Juhl-Larsen *et al.* 2012]
- **parametric** modal transition systems: [Beneš-Křetínský-Larsen *et al.* 2011]
- for **deadlock** equivalence: [Bujtor-Sorokin-Vogler 2015]

[UF-Legay SOFSEM 2017]: The Linear-Time–Branching-Time Spectrum of Specification Theories

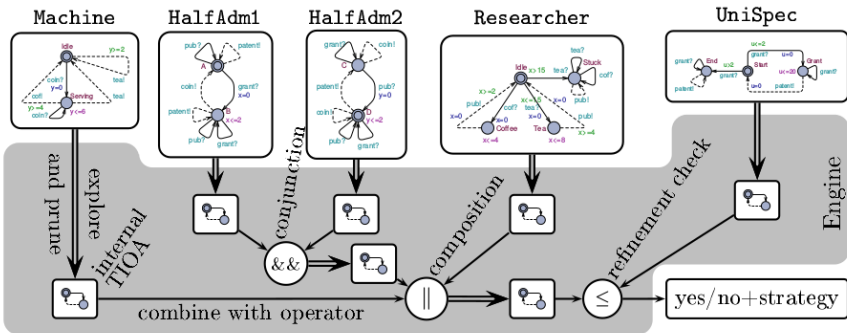


Specification Theories for Real-Time Systems

Timed input-output automata:

- [David-Larsen-Legay *et al.*: Real-time specifications, STTT 2015], [David-Larsen-Legay *et al.*: Compositional verification of real-time systems using ECDAR, STTT 2012]
- complete, with quotient, but without disjunction
- only for **deterministic** specifications
- tool support: **ECDAR** / **UPPAAL TiGa** (Aalborg)
- some work on **robustness** and **implementability**: [Larsen-Legay-Traonouez *et al.*: Robust synthesis for real-time systems, TCS 2014]

Timed Input-Output Automata



Specification Theories for Real-Time Systems, contd.

Modal **event-clock** specifications:

- [Bertrand-Legay-Pinchinat *et al.*: Modal event-clock specifications for timed component-based design, SCP 2012]
- complete, with quotient, but without disjunction
- only for **deterministic** specifications
- some work on **robustness**: [UF-Legay 2012]

Synchronous time-triggered interface theories:

- [Delahaye-UF-Henzinger *et al.* 2012]
- no quotient, no real conjunction, no implementation
- relation to **BIP** (Grenoble)

Specification Theories for Probabilistic (Timed) Systems

Abstract probabilistic automata:

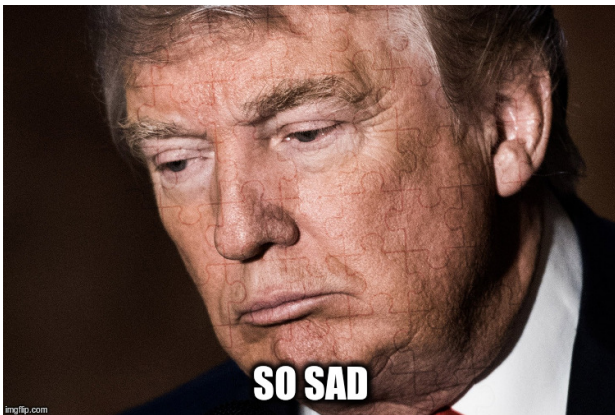
- [Delahaye-Katoen-Larsen *et al.*: Abstract probabilistic automata, I&C 2013], [Delahaye-UF-Larsen *et al.* 2014]
- no quotient, no disjunction, toy implementation

Abstract probabilistic event-clock automata:

- [Han-Krause-Kwiatkowska *et al.* 2013]
- no quotient, no disjunction, no implementation, other problems

Specification Theories for Hybrid Systems

Specification Theories for Hybrid Systems



Interfaces and Contracts

Modal interface automata

- [Lüttgen-Vogler: Modal interface automata, LMCS 2013]
- interface automata: [de Alfaro-Henzinger 2001]
- inputs vs outputs
- complete, without quotient

From specifications to **contracts**:

- [Bauer-David-Hennicker *et al.* 2012]
- complete specification theory \implies contract theory
- in a timed setting: [Le-Passerone-UF *et al.*: A tag contract framework for modeling heterogeneous systems, SCP 2016]

Robust Specification Theories

Definition (recall)

A specification theory $(\text{Spec}, \leq, \chi)$ for (Mod, \sim) is **nice** if (Spec, \leq) forms a bounded distributive lattice up to $\equiv = \leq \cap \geq$.

- for **robustness**: replace \sim by **pseudometric** d_{Mod}
- (such that $d_{\text{Mod}}(\mathcal{M}_1, \mathcal{M}_2) = 0$ iff $\mathcal{M}_1 \sim \mathcal{M}_2$)
- replace \leq by **non-symmetric** pseudometric d (“**hemimetric**”)
- (d_{Mod} and d are related via χ)
- instead of $\mathcal{M} \models \mathcal{S}_1 \ \& \ \mathcal{S}_1 \leq \mathcal{S}_2 \implies \mathcal{M} \models \mathcal{S}_2$,
want **$d(\mathcal{M}, \mathcal{S}_1) + d(\mathcal{S}_1, \mathcal{S}_2) \geq d(\mathcal{M}, \mathcal{S}_2)$**
- $d(\mathcal{S}, \mathcal{S}_1 \wedge \mathcal{S}_2) = \max(d(\mathcal{S}, \mathcal{S}_1), d(\mathcal{S}, \mathcal{S}_2), \infty)$
- $d(\mathcal{S}_1 \vee \mathcal{S}_2, \mathcal{S}) = \max(d(\mathcal{S}_1, \mathcal{S}), d(\mathcal{S}_2, \mathcal{S}), \infty)$

Robust Specification Theories, contd.

Definition (recall)

A **complete specification theory** for (Mod, \sim) is $(\text{Spec}, \leq, \parallel, \chi)$ such that $(\text{Spec}, \leq, \chi)$ is a specification theory for (Mod, \sim) and $(\text{Spec}, \leq, \parallel)$ forms a bounded distribute commutative residuated lattice up to \equiv .

- for **independent implementability**, want **uniform continuity** for \parallel :
a function $C : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ such that we can
replace $\mathcal{S}_1 \leq \mathcal{S}_3$ & $\mathcal{S}_2 \leq \mathcal{S}_4 \implies \mathcal{S}_1 \parallel \mathcal{S}_2 \leq \mathcal{S}_3 \parallel \mathcal{S}_4$
with $C(d(\mathcal{S}_1, \mathcal{S}_3), d(\mathcal{S}_2, \mathcal{S}_4)) \geq d(\mathcal{S}_1 \parallel \mathcal{S}_2, \mathcal{S}_3 \parallel \mathcal{S}_4)$
- for **quotient**, instead of $\mathcal{S}_1 \parallel \mathcal{S}_2 \leq \mathcal{S}_3 \iff \mathcal{S}_2 \leq \mathcal{S}_3 / \mathcal{S}_1$
want $d(\mathcal{S}_1 \parallel \mathcal{S}_2, \mathcal{S}_3) = d(\mathcal{S}_2, \mathcal{S}_3 / \mathcal{S}_1)$
- [UF-Legay TCS 2014], [UF-Legay Acta Inf. 2014],
[UF-Křetínský-Legay *et al.* 2014]

Conclusion?

- **incrementality:** $\mathcal{M} \models \mathcal{S}_1 \ \& \ \mathcal{S}_1 \leq \mathcal{S}_2 \implies \mathcal{M} \models \mathcal{S}_2$
- **conjunction:** $\mathcal{M} \models \mathcal{S}_1 \ \& \ \mathcal{M} \models \mathcal{S}_2 \iff \mathcal{M} \models \mathcal{S}_1 \wedge \mathcal{S}_2$
- **disjunction:** $\mathcal{M} \models \mathcal{S}_1 \ \text{or} \ \mathcal{M} \models \mathcal{S}_2 \iff \mathcal{M} \models \mathcal{S}_1 \vee \mathcal{S}_2$
- **compositionality:** $\mathcal{M}_1 \models \mathcal{S}_1 \ \& \ \mathcal{M}_2 \models \mathcal{S}_2 \implies \mathcal{M}_1 \parallel \mathcal{M}_2 \models \mathcal{S}_1 \parallel \mathcal{S}_2$
- **quotient:** $\mathcal{M}_1 \models \mathcal{S}_1 \ \& \ \mathcal{M}_2 \models \mathcal{S} / \mathcal{S}_1 \implies \mathcal{M}_1 \parallel \mathcal{M}_2 \models \mathcal{S}$
- **safety properties**
- Are these all the properties we want?
- Also need robustness
- Long way
 - from acceptance automata
 - to hybrid systems
 - to industry ...