



# Energy Büchi Problems

ADADL 2023

Sven Dziadek   **Uli Fahrenberg**   Philipp Schlehuber-Caissier

LRE, EPITA, France



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- Timed automata
- Büchi condition



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energy always within bound  $[0, b]$

weak upper bound  $b$





## Remember, Remember, the 15 September

- ... 2008
- Bouyer, F., Larsen, Markey, Srba: *Infinite Runs in Weighted Timed Automata with Energy Constraints*, FORMATS 2008
- Dziadek, F., Schlehuber: *Energy Büchi Problems*, FM 2023:
  - ▶ extend to Büchi conditions
  - ▶ fix problems
  - ▶ **implement everything**: TChecker + Spot



# Weighted Timed Büchi Automata



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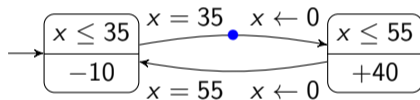
## Weighted Timed Büchi Automata

- generalized Büchi acceptance on transitions
- (only) locations are weighted

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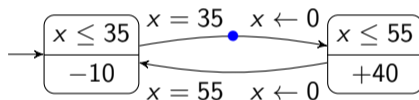
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# Weighted Timed Büchi Automata

## Weighted Timed Büchi Automata

- generalized Büchi acceptance on transitions
- (only) locations are weighted



Note: we only handle **one clock**

Energy problems **undecidable** for **four** clocks (Bouyer, Larsen, Markey 2014)

**open** for **two** or **three** clocks

## Corner-Point Abstraction

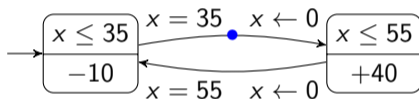
One-clock timed automaton  $\rightarrow$  untimed automaton

- TChecker computes the zone graph
- compute corner-point abstraction (Behrmann, Fehnker, Hune et al. 2001)
- Zeno-exclusion

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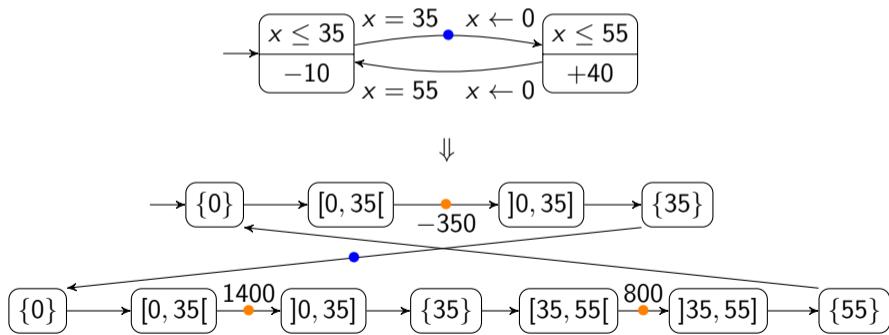
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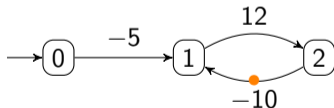
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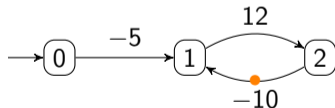
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Feasible with  $c \geq 5$  and  $b \geq 10$ .

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Our solution: Search for **strongly connected components**  
and **modify BF** for “energy positive” loops

## Our Algorithm

Take a weighted Büchi automaton:

- find strongly connected components (SCC)  
(we use Couvreur)
- degeneralize SCCs  
(produces Büchi accepting **back edges**)
- with modified Bellman-Ford  
search for feasible lassos:
  - ▶ on original graph for maximal **prefix** energy
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Note: Energy and Büchi condition cannot be fully separated

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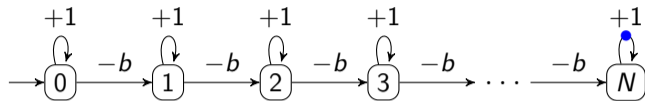
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#mod	#states	to cpa [s]	sol [s]
1	25	0.01	0.00
3	90	0.03	0.02
5	293	0.06	0.24
7	1012	0.19	3.24
9	3759	0.89	59.52
10	7377	1.87	261.38
11	14582	4.37	1194.81

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## Modified BF: Challenges

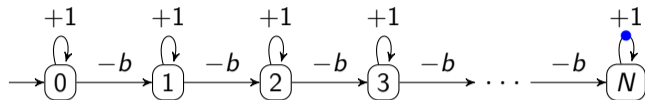
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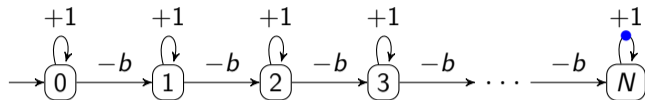
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Get weak upper bound  $b$  out of complexity

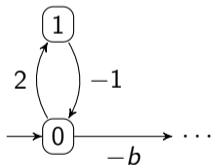
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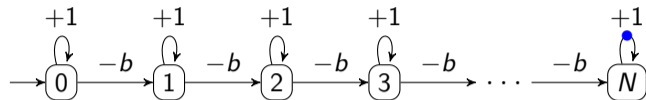
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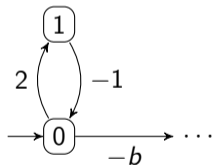
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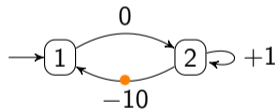


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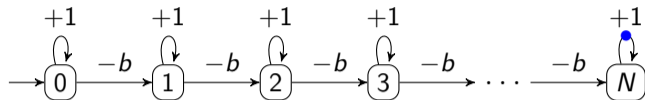


### Example (for $c = 30, b = 30$ )



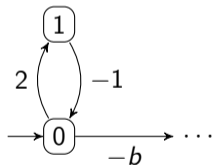
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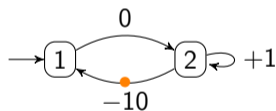


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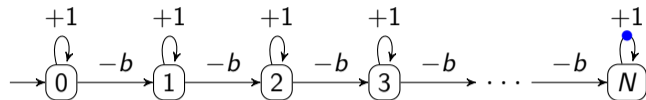
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Max energy:  $30$

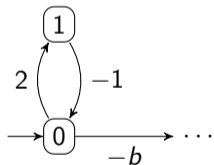
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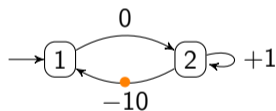


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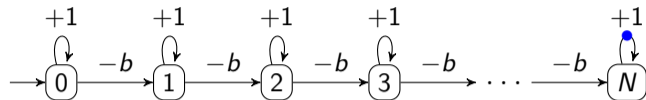
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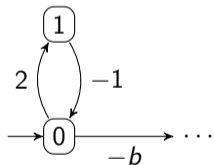
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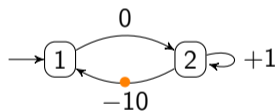


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Max energy:

30  
20  
30

# Conclusion



## Results on Energy Büchi problems

1. *Weighted Büchi automata*
  - Modified Bellman-Ford with Couvreur's algorithm
2. *One-clock weighted timed Büchi automata*
  - Reduce to **1.** using corner-point abstraction

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## Future Work

- edge weights
  - ▶ Bouyer, F., Larsen, Markey: *Timed automata with observers under energy constraints*, HSCC 2010
- more realistic battery model
  - ▶ Boker, Henzinger, Radhakrishna: *Battery Transition Systems*, POPL 2014
- **parametric problem**: synthesize  $b$  and/or  $c$ 
  - ▶ F., Juhl, Larsen, Srba: *Energy Games in Multiweighted Automata*, ICTAC 2011
  - ▶ (in some cases that's easier!)
- **implement everything!**

Backup



