



Energy Büchi Problems

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Energy Büchi Problem



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- Timed automata
- Büchi condition



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 - ▶ negative weight: **consumption** of energy
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Does a Büchi accepted feasible run exist?

energy always within bound $[0, b]$

weak upper bound b



Remember, Remember, the 15 September

- ... 2008
- Bouyer, F., Larsen, Markey, Srba: *Infinite Runs in Weighted Timed Automata with Energy Constraints*, FORMATS 2008
- Dziadek, F., Schlehuber: *Energy Büchi Problems*, FM 2023:
 - ▶ extend to Büchi conditions
 - ▶ fix problems
 - ▶ **implement everything**: TChecker + Spot



Weighted Timed Büchi Automata



Weighted Timed Büchi Automata

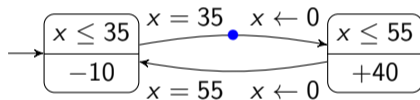
Weighted Timed Büchi Automata

- generalized Büchi acceptance on transitions
- (only) locations are weighted

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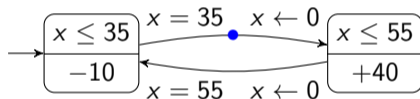
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Weighted Timed Büchi Automata

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Note: we only handle **one clock**

Energy problems **undecidable** for **four** clocks (Bouyer, Larsen, Markey 2014)
open for **two** or **three** clocks

Corner-Point Abstraction

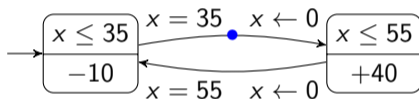
One-clock timed automaton \rightarrow untimed automaton

- TChecker computes the zone graph
- compute corner-point abstraction (Behrmann, Fehnker, Hune et al. 2001)
- Zeno-exclusion

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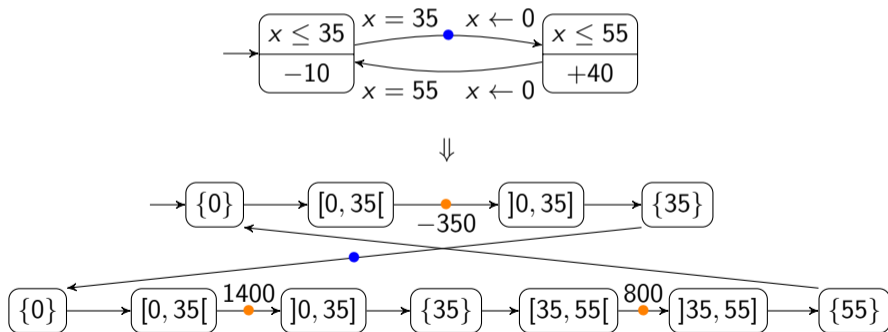
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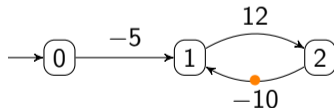
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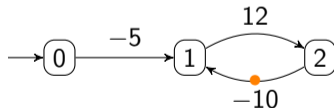
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Feasible with $c \geq 5$ and $b \geq 10$.

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Our solution: Search for **strongly connected components**
and **modify BF** for “energy positive” loops

Our Algorithm

Take a weighted Büchi automaton:

- find strongly connected components (SCC)
(we use Couvreur)
- degeneralize SCCs
(produces Büchi accepting **back edges**)
- with modified Bellman-Ford
search for feasible lassos:
 - ▶ on original graph for maximal **prefix** energy
 - ▶ in SCCs for **non-negative cycles** including a **back edge**

Note: Energy and Büchi condition cannot be fully separated

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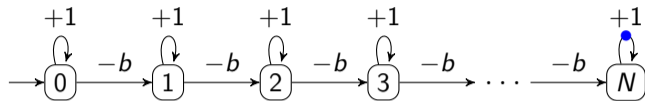
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#mod	#states	to cpa [s]	sol [s]
1	25	0.01	0.00
3	90	0.03	0.02
5	293	0.06	0.24
7	1012	0.19	3.24
9	3759	0.89	59.52
10	7377	1.87	261.38
11	14582	4.37	1194.81

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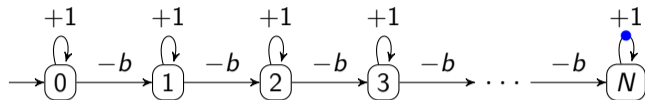
Modified BF: Challenges

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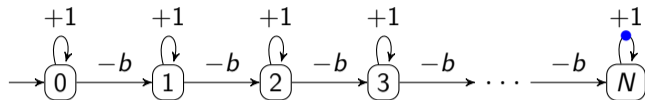
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Get weak upper bound b out of complexity

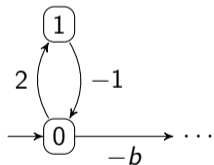
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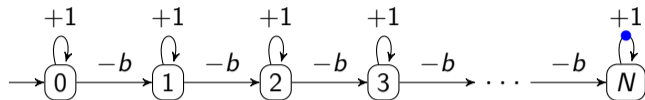
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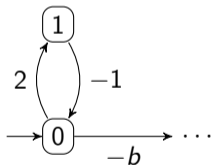
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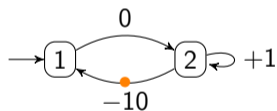


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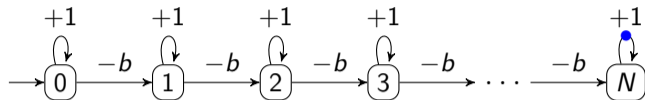


Example (for $c = 30, b = 30$)



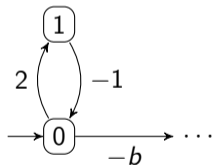
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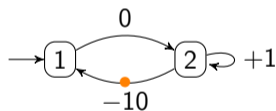


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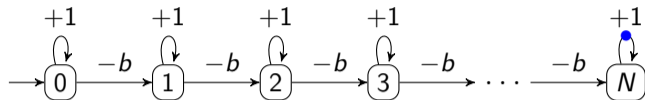
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Max energy: 30

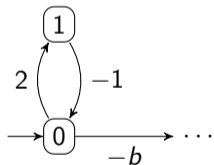
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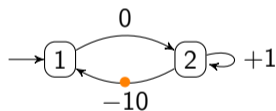


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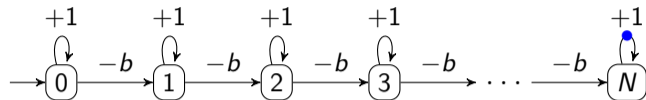
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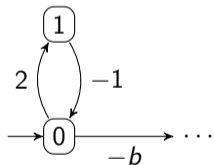
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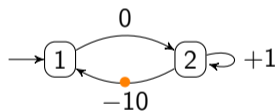


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Max energy:

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20 30

Conclusion



Results on Energy Büchi problems

1. *Weighted Büchi automata*
 - Modified Bellman-Ford with Couvreur's algorithm
2. *One-clock weighted timed Büchi automata*
 - Reduce to **1.** using corner-point abstraction

All algorithms are implemented using TChecker and Spot



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Future Work

- edge weights
 - ▶ Bouyer, F., Larsen, Markey: *Timed automata with observers under energy constraints*, HSCC 2010
- more realistic battery model
 - ▶ Boker, Henzinger, Radhakrishna: *Battery Transition Systems*, POPL 2014
- **parametric problem**: synthesize b and/or c
 - ▶ F., Juhl, Larsen, Srba: *Energy Games in Multiweighted Automata*, ICTAC 2011
 - ▶ (in some cases that's easier!)
- **implement everything!**

Backup



