WP1: Languages, Bisimulations and Logics for Higher-Dimensional Automata

ANR CHAAT kickoff

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Overview of WP1

WP1: Languages, Bisimulations and Logics for Higher-Dimensional Automata

- **WP1.1.** Language theory of finite HDAs
- **WP1.2.** HDAs on infinite pomsets
- **WP1.3.** Bisimulations and logics for concurrency and action refinement
- **WP1.4.** Active learning of models for concurrency

State of the art (non-exhaustive)

How to produce an HDA paper

Recipe to create your own HDA paper

- Pick your favourite theorem of automata theory.
- Check whether it still works for HDAs.
- Often, yes, but with a twist!

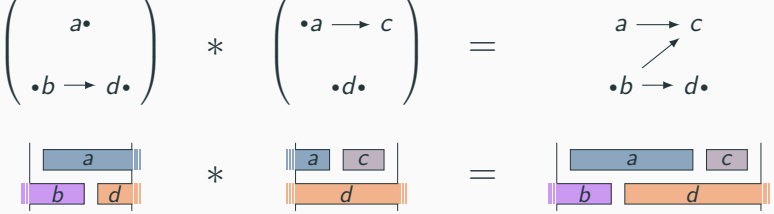
Kleene Theorem

Kleene Theorem for Higher-Dimensional Automata (2024).
Uli Fahrenberg, Christian Johansen, Georg Struth, Krzysztof Ziemiański.

Kleene theorem for HDAs

A language of pomsets is regular if and only if it is rational.

Twist: we need to redefine concatenation. Instead, use pomsets with interfaces, and gluing!



Myhill-Nerode Theorem

Myhill-Nerode Theorem for Higher-Dimensional Automata (2024).
Uli Fahrenberg, Krzysztof Ziemiański.

Myhill-Nerode theorem for HDAs

Let L be a language of pomsets that is downward-closed by subsumption.
Then L is regular iff the set of prefix quotients $\{P\backslash L \mid P \in \mathrm{iiPoms}\}$ is finite.

Twist: several technical difficulties in the ($\Leftarrow$) direction.

- Cells of the constructed HDA are equivalence classes for the strong equivalence relation: $P \simeq Q$ iff for all A in the terminating interface, $(P - A)\backslash L = (Q - A)\backslash L$.
- Additional subsidiary cells that do not affect the language, to define some lower faces.

Büchi Theorem

Büchi-Elgot-Trakhtenbrot Theorem for Higher-Dimensional Automata (2025).
Amazigh Amrane, Hugo Bazille, Emily Clement, Uli Fahrenberg, Marie Fortin, Krzysztof Ziemiański.

Büchi-Elgot-Trakhtenbrot theorem for HDAs

Let $L \subseteq$ iiPoms be a language of pomsets.

- If L is regular, then it is MSO-definable.
- If L is MSO-definable, then $L_{\leq k}{\downarrow}$ is regular for all $k \in \mathbb{N}$.

Twist: HDAs can only recognize languages that are downward-closed w.r.t. subsumption, and have bounded dimension. MSO logic has no such restriction.

Closure and decision properties

Closure and Decision Properties for Higher-Dimensional Automata (2025).
Amazigh Amrane, Hugo Bazille, Uli Fahrenberg, Krzysztof Ziemiański.

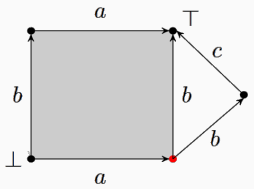
Various properties of HDAs

Higher-dimensional automata have the following properties (and more):

- Pumping lemma
- Decidability of inclusion and emptiness
- Closure under intersection and under bounded-width complement

Twist:

- HDAs are not determinizable.
- They are not closed under complement.



Kamp Theorem

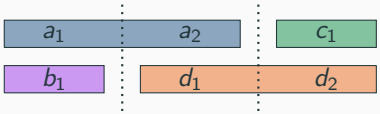
Kamp Theorem for Higher Dimensional Automata (2026).
Emily Clement, Enzo Erlich, J.L.

Kamp theorem for HDAs

Let $L \subseteq$ iiPoms be a language of pomsets of bounded dimension k.
Then L if recognized by an FO formula iff L is recognized by an $\mathrm{LTL}_{\mathrm{Poms}}$ formula.

Twist: the LTL logic is evaluated on sub-events.

- Next and Until operators are non-standard.
- Other variants of the logic may be possible.



WP1: Languages, Bisimulations and Logics for HDA

WP1.1: Language theory of finite HDAs

There are still many things to be done, e.g.:

- History-deterministic HDAs (or even "state-deterministic"?)
- Star-free regular expressions
- Counter-free HDAs
- Other variants of LTL on pomsets

More generally, develop a language theory for:

- Non-downward-closed languages $\longrightarrow$ Partial HDAs, ST-automata (too easy?).
- Pomsets without event order $\longrightarrow$ Symmetric HDAs, logics with counting operators.
- Non-interval pomsets $\longrightarrow$ Krzysztof knows how to do this (use track objects).

PhD student: Enzo Erlich (EPITA & IRIF).

WP1.2: HDAs on infinite pomsets

Higher-dimensional automata: Extension to infinite tracks (2025).
Luc Passemard, Amazigh Amrane, Uli Fahrenberg

This gets complicated:

- ω-HDA languages are not closed under subsumption
- Muller acceptance condition is more expressive than Büchi

Even more complicated: infinite parallelism.

Irrationality of Process Replication for Higher-Dimensional Automata (2024).
Thomas Baronner, Henning Basold, Márton Hablicsek

PhD student: Jakub Hajdas (Télécom SudParis).

WP1.3: Bisimulations and logics for concurrency and action refinement

Idea: view HDAs not as recognizers, but as models for concurrent systems.

- What are reasonable notions of bisimulations?
- What kind of behavioral semantics do they capture?
- What are the corresponding logics?

> *Bisimulations and Modal Logics for Higher Dimensional Automata (2026).*
> Safa Zouari, Rob van Glabbeek, Krzysztof Ziemianski

We should also study action refinement: what happens when we change the granularity of the model description? E.g. action a becomes $a_1; a_2; a_3$.

WP1.4: Active learning of models for concurrency

Goal: infer a formal model of a black-box system using

- Membership queries
- Equivalence queries

Some preliminary work has been done:

> *Active Learning Techniques for Pomset Recognizers (2026).*
> Adrien Pommellet, Amazigh Amrane, Edgar Delaporte, Geoffroy Du Prey, Oscar Peyron

but still needs to be properly extended to HDAs!

PhD student: Safia Tehami (LMF/LRE).

Summary

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- **WP1.3.** Bisimulations and logics for concurrency and action refinement
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- **Bonus task:** write a book!