Hyper Partial Order Logic

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Motivations

$\frac{\text{Non Interference}}{\Sigma = \Sigma_{low} \uplus \Sigma_h}$



$$\mathcal{L}(\mathcal{O}_{low}(S)) \subseteq \mathcal{L}(\mathcal{O}_{low}(S \backslash \Sigma_h))$$

$$\forall \rho_1.h.\rho_1', \exists \rho_2 \in \sigma_{low}^*, \\ \mathcal{O}_{low}(\rho_1.h.\rho_1') = \mathcal{O}_{low}(\rho_2)$$

 \underline{Pb} : cannot be expressed with a LTL, CTL property.

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Mantel's framework

Comparison of language closures (projections, morphisms,...)

[Mantel00] [D' Souza11, 16]

$$\begin{array}{ll} & \textit{op}_1(\mathcal{L}(S)) \subseteq \textit{op}_2(\mathcal{L}(S)) \\ \wedge & \textit{op}_3(\mathcal{L}(S)) \subseteq \textit{op}_4(\mathcal{L}(S)) \\ \wedge & \dots \end{array}$$

Hyperproperties

Properties of sets of traces [Clarkson14, Schneider10]

$$\mathcal{L}(\mathcal{O}_{low}(S)) \subseteq \mathcal{L}(\mathcal{O}_{low}(S \backslash \Sigma_h))$$

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[Mantel00] [D' Souza11, 16]

 $\underset{\text{(projec-}}{\underline{\text{Alur's framework}}} : \underline{\text{CTL}} \sim$

[Alur07]

ec- CTL + a relation on equivalent events

$$\begin{array}{l}
op_1(\mathcal{L}(S)) \subseteq op_2(\mathcal{L}(S)) \\
\wedge op_3(\mathcal{L}(S)) \subseteq op_4(\mathcal{L}(S))
\end{array}$$

Hyperproperties

Properties of sets of traces

$$[Clarkson14, Schneider10]$$
 $\mathcal{L}(\mathcal{O}_{low}(S)) \subseteq \mathcal{L}(\mathcal{O}_{low}(S \setminus \Sigma_h))$



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<u>Pb</u>: cannot be expressed with a LTL, CTL property.

Local Logics

\mathcal{O}_{low} projection on events with label in $\{a,b\}$.



Interleaved setting:



Formula of the form $\phi ::= X(p_a \wedge Xp_b)$ does not characterize O_1

Partial order setting

$$O, e \models \lambda(e) = a \land \exists f, e \prec f, \lambda(f) = b$$

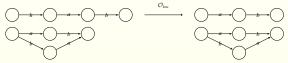
Address the shape of causal ordering among events in a single partial order! LD_0 [Meenakshi04], TCL^- [Peled00],...

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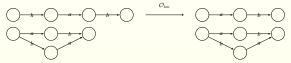
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$$O, e \models \lambda(e) = a \land \exists f, e \prec f, \lambda(f) = b$$

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Outline

HYPOL : an Hyper Partial Order Logic

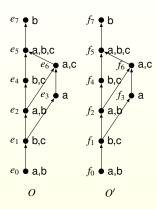
Part 1: Hypol

- Partial orders, template
- Partial observations
- Hypol : Syntax & Semantics
- Satisfiability
- Example : causal non-interference

Part 2: Model Checking on Petri nets processes

- Unfolding & processes
- A grammar for unfolding
- Execution graphs
- From Hypol to MSO
- Observable nets

Partial Orders, templates



LPO over Σ

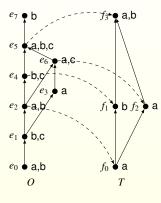
$$O = (E, \leq, \lambda)$$

- *E* is a set of events.
- $\leq \subseteq E \times E$ partial order,
- $\lambda: E \to 2^{\Sigma}$ labeling

Definition: Isomorphism

$$O=(E,\leq,\lambda)$$
 and $O'=(E',\leq',\lambda')$ are isomorphic $(O\equiv O')$ iff $\exists h:E\to E'$ such that $e\leq e'\Longleftrightarrow h(e)\leq' h(e')$ and $\lambda(e)=\lambda'(h(e)).$

Partial Orders, templates



LPO over Σ

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Template Matching

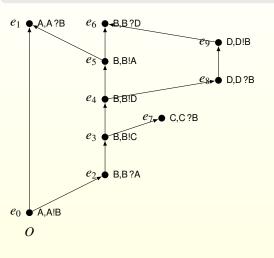
 $O=(E,\leq,\lambda)$ and $T=(E_T,\leq_T,\lambda_T)$ O matches T iff $\exists h\subseteq E,h:H\to E_T$ such that :

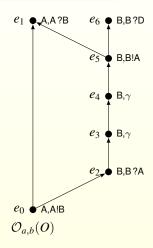
- $\bullet \ \lambda_T(h(e)) \subseteq \lambda(e),$
- $e <_T e'$ implies $h^{-1}(e) < h^{-1}(e')$.

Partial Observations

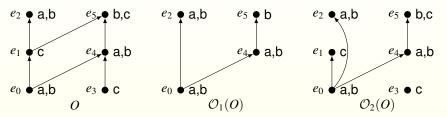
Observation function

mapping $\mathcal{O}:\mathcal{L}PO(\Sigma)\to\mathcal{L}PO(\Sigma'),$ representing the visible part of the system.





Observation : examples



 $\mathcal{O}_1(O)$: projection on events that carry label a or b,

 $\mathcal{O}_2(\mathit{O})$: restriction of \leq to events with an a

Main idea: model the observation power of an intruder.

Hypol: Syntax & Semantics

- A, Σ atomic propositions
- \mathcal{T} finite set of templates over A,
- Obs finite set of observation functions

$$\phi ::= true \mid \neg \phi \mid \phi_1 \lor \phi_2 \mid match(\mathcal{O}, T, f) \mid EX_{D,\mathcal{O}} \phi \mid EX_{\equiv,\mathcal{O}} \phi \mid \phi_1 EU_{D,\mathcal{O}} \phi_2 \mid EG_{D,\mathcal{O}} \phi$$

where $D \subseteq A$, $T \in \mathcal{T}$, f is an event of T, and $\mathcal{O} \in \mathcal{O}bs$

Semantics

Evaluation of formulas

Formulas are evaluated over a set ${\mathcal W}$ of LPOs over $\Sigma,$

$$\mathcal{W}$$
 satisfies ϕ iff $\exists O = (E, \leq, \lambda) \in W, e \in min(O)$,

$$O, e \models \phi$$

Satisfiability

A formula ϕ is satisfiable iff there exists an universe $\mathcal W$ such that $\mathcal W\models\phi$

Satisfiability problem : Given ϕ , is it satisfiable by some universe \mathcal{W} ?

Model checking

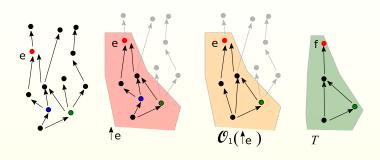
A model M satisfies a formula ϕ iff the universe \mathcal{W}_{M} of its executions satisfies ϕ

Model checking problem : Given M, ϕ , does $\mathcal{W}_M \models \phi$?

Semantics : Matching

 $O, e \models match(\mathcal{O}_1, T, f)$

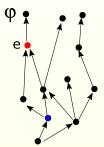
iff



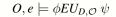
- one can match T in the observation $\mathcal{O}_1(\downarrow e)$ (causal past of e).
- ullet with at least a witness mapping $h_{e,f}$ associating f with e

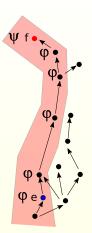
Semantics : $EX_{D,\mathcal{O}}$ and $EU_{D,\mathcal{O}}$

$$O, e \models EX_{D,\mathcal{O}} \phi$$



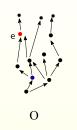
The next observed event satisfies ϕ

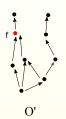




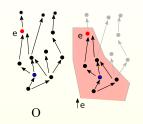
There exists an event in the future that satisfies ϕ

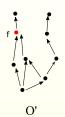
Semantics : $EX_{\equiv,\mathcal{O}}$



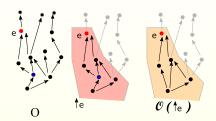


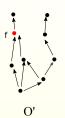
$$O, e \models \mathit{EX}_{\equiv,\mathcal{O}} \phi$$



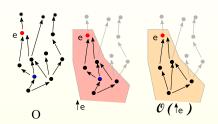


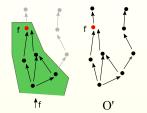
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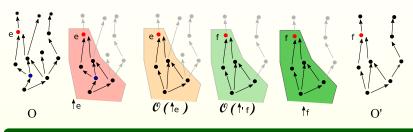


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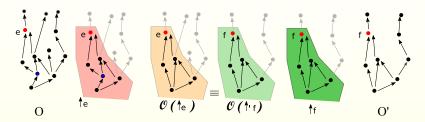




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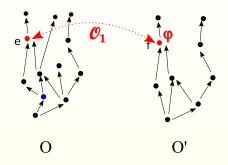


$$O, e \models \mathit{EX}_{\equiv,\mathcal{O}} \phi$$



$$O, e \models \mathit{EX}_{\equiv,\mathcal{O}} \phi$$

Semantics : $EX_{\equiv,\mathcal{O}}$



$O, e \models EX_{\equiv,\mathcal{O}} \phi$

There exists another order O' in W and an event f such that

- \bullet $O', f \models \phi$
- ullet ${\mathcal O}$ cannot distinguish the causal past of e and f

$$\mathcal{O}(\uparrow e) \equiv \mathcal{O}(\uparrow e)$$

An example: causal Non-Interference

Let
$$\Sigma = \Sigma_{high} \uplus \Sigma_{low}$$
 with $\Sigma_{high} = \{h\}$ and $\Sigma_{low} = \{a, b\}$

 \mathcal{O}_{low} projection of LPOs on events with label in Σ_{low} .

Causal Non-Interference

$$\begin{split} T_{\mathsf{h} \leq a} &= \bullet^h \longrightarrow \bullet^a \\ Pred_h &::= \bigvee_{a \in \Sigma} match(\mathcal{O}_{\mathsf{h},a}, T_{\mathsf{h} \leq a}) \\ \phi_{\mathit{CNI}} &::= AG_{\Sigma,id} \left(\lambda_{\in \Sigma_{\mathit{high}}} \lor Pred_h \right) \Longrightarrow EX_{\equiv, \mathcal{O}_{\mathit{low}}} (\lambda_{\not \in \Sigma_{\mathit{high}}} \land \neg Pred_h)) \end{split}$$

If a system satisfies ϕ_{CNI} , then an intruder with observation capacity \mathcal{O}_{low} cannot differentiate runs with/without h.

In particular, a system with behaviors $\mathcal{W}=\{O_1,O_2\}$ does not satisfy $\phi_{\textit{CNI}}$ and is not secure

Satisfiability

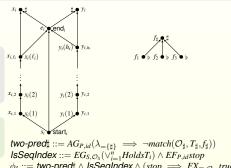
Theorem

Satisfaibility of Hypol is undecidable

A PCP encoding:

$$I = \{(x_1, y_1), \dots, (x_n, y_n)\}$$
$$(x_i, y_i) \text{ pair of words in } A^*.$$

$$\exists i_1 \dots i_k$$
 such that $x_{i_1} \dots x_{i_k} = y_{i_1} \dots y_{i_k}$?



 $\phi_I ::= two\text{-pred}\sharp \wedge lsSeqIndex \wedge (stop \implies EX_{\equiv \mathcal{O}_{rel}}true)$

PCP instance *I* has a solution if $\exists O, e$ such that $O, e \models \phi_I$

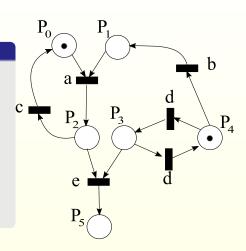
Part 2 : Model Checking on Petri nets Processes

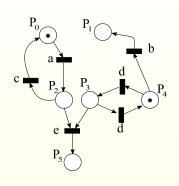
Petri nets

Definition

A *labeled Petri net* is a tuple $\mathcal{N} = (P, T, F, M_0)$

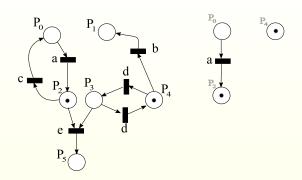
- *P* : set of places,
- T set of transitions
- $F \subseteq P \times T \cup T \times P$ flow relation,
- $M_0 \in \mathbb{N}^P$ is the initial marking.
- $\bullet \ \lambda: T \to \Sigma$



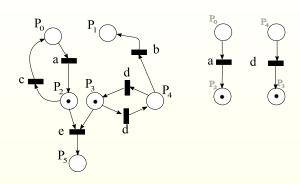


$$\mathcal{N}$$

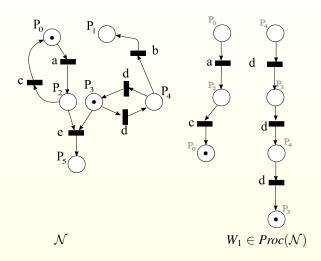
$$\mathit{W}_1 \in \mathit{PR}(\mathcal{N})$$

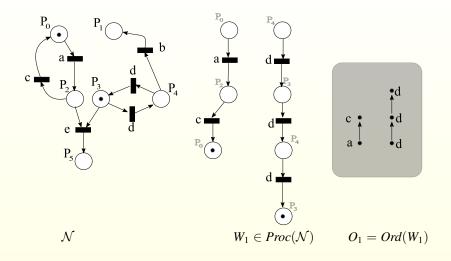


$$\mathcal{N}$$



$$\mathcal{N}$$





Model checking Hypol on Petri nets

Let \mathcal{N} be a Petri net, $PR(\mathcal{N})$ its set of processes Let ϕ be an hypol formula :

$$\mathcal{N} \models \phi$$
 iff $\exists O \in Ord(PR(\mathcal{N})), e \in min(O)$ such that $O, e \models \phi$

Unfortunately

Undecidability

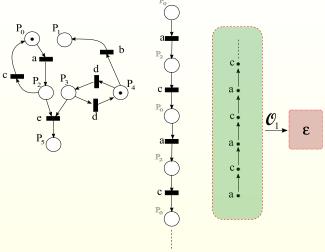
Model checking of Hypol properties for (safe) Petri nets is undecidable.

Why : PCP encoding. For every instance I of PCP, can build a net \mathcal{N}_I such that $\mathcal{N}_I \models \phi_I$ iff this instance of PCP has a solution.

Definition

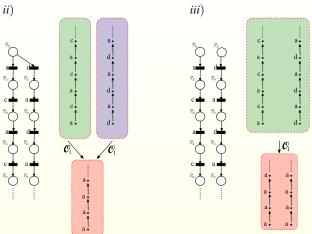
 \mathcal{N} is *observable* (wrt $\mathcal{O}_1, \dots \mathcal{O}_k$) iff

- *i*) $\forall \mathcal{O}_i$, every cyclic behavior produces something observable by \mathcal{O}_i at every iteration,
- *ii*) $\forall \mathcal{O}_i$, choices eventually appear in observation after k_c steps,
- *iii*) $\forall \mathcal{O}_i$, there exists a bound on the size of parallel threads which have identical observation



 \mathcal{O}_1 : projection on events with labels in $\{b,d,e\}$.

 $i): \mathcal{N}$ does not remain unobservable forever



 \mathcal{O}_1 : projection on events with label a.

$$(i) + (ii) + (iii) \implies$$
 an event e is always equivalent to a bounded number of events in a bounded past / parallel part

Model checking of Hypol

Theorem

Hypol model-checking is decidable for observable nets.

- ullet show that processes of a nets can be seen as a regular graph $\mathcal{G}^\omega_\mathcal{N}$
- isomorphism up to observation \mathcal{O}_i can be encoded as an additional relation $\stackrel{i}{\longrightarrow}$: gives a new (non regular) graph $G_{\mathcal{N}}$
- Show that Hypol properties of safe nets can be encoded as MSO properties
- Identify a class of K-layered nets where G_N is regular Hypol decidable on this class!
- Show that observable nets belong to this class

Branching processes

Branching processes "unfold" Petri nets

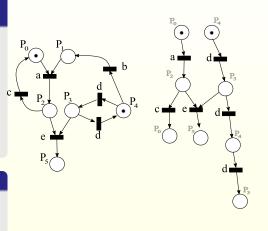
Definition (Branching Process)

A branching process of $\mathcal{N} = (P, T, F, M_0, \lambda)$ is a triple $BR = (ON, \mu, \lambda')$

- $ON = (B, E, \hat{F}, Cut_0)$ is an occurrence net,
- μ is a homomorphism and $\forall e \in E, \lambda'(e) = \lambda(\mu(e)).$

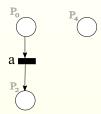
Definition (Unfolding)

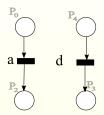
The *unfolding* of \mathcal{N} , $\mathcal{U}(\mathcal{N})$, is the maximal branching process.

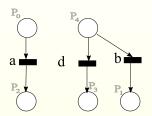


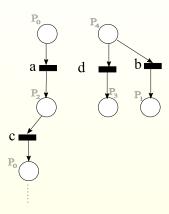
 $\mathcal{U}(\mathcal{N})$ can be seen as the union of all processes of \mathcal{N}

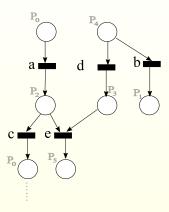


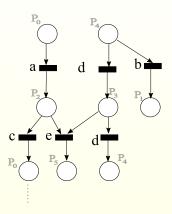


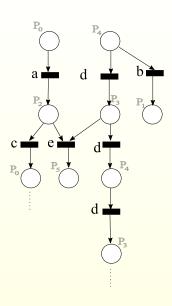


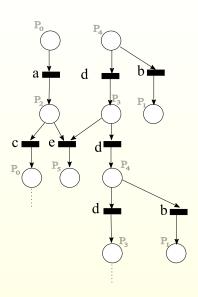


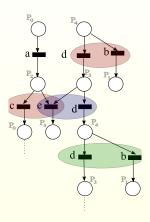












Conflicts

Two events e, f are conflicting if

- they are not causally related
- they have a common place in their past

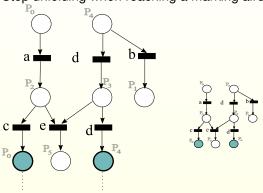
Processes in $\mathcal{U}_{\mathcal{N}}$

- ullet $\mathcal{U}_{\mathcal{N}}$ is a graph
- Processes of $\mathcal N$ are projections of $\mathcal U_{\mathcal N}$ on maximal conflict-free sets of events & conditions

A grammar for unfolding

Idea of the construction:

Stop unfolding when reaching a marking already drawn

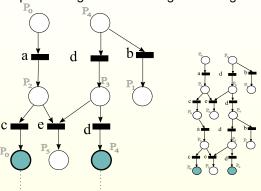


Close to Complete Finite Prefixes [McMillan95, Esparza02]

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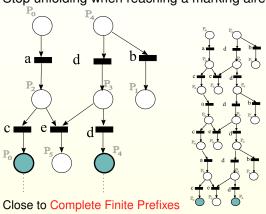


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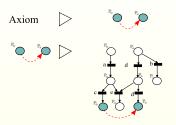


[McMillan95, Esparza02]

Graph Grammars

Theorem

One can effectively build a graph grammar $\mathcal{G}_{\mathcal{N}}$ that generates $\mathcal{U}_{\mathcal{N}}$



Interesting...

Graphs generated by graph grammars have bounded treewidth

- size of the largest vertex set in a tree decomposition of the graph
- nb. colors needed to generate a graph with a simple graph algebra

MSO is decidable for graph grammars

Idea: translate ϕ to an equivalent MSO formula but ...

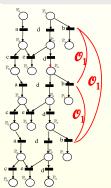
Isomorphism cannot be expressed in MSO.

Execution graph

Execution Graph

An unfolding, plus explicit representation of isomorphisms

$$G_{\mathcal{U}_{\mathcal{N}}} = \mathcal{U}_{\mathcal{N}} \uplus \{e \xrightarrow{i} f \mid \mathcal{O}_{i}(\downarrow e) \equiv \mathcal{O}_{i}(\downarrow f)\}$$



Proposition

There exist labeled safe Petri nets and observation functions whose execution graphs are not of bounded treewidth

 \mathcal{O}_1 : erases occurrences of d

From Hypol to MSO

Theorem

For every Hypol formula ϕ and every safe Petri net \mathcal{N} , there exists an MSO formula ψ such that $\mathcal{N} \models \phi$ iff $G_{\mathcal{U}_{\mathcal{N}}} \models \psi$

Proof idea:

- e < f, $e \le f$, $x \le_{\mathcal{O}} y$ expressible as an MSO property of $G_{\mathcal{U}_{\mathcal{N}}}$.
- $\mathcal{O}_i(\downarrow e) \equiv \mathcal{O}_i(\downarrow f)$ is a simple relation $e \stackrel{i}{\longrightarrow} f$

Then inductive construction.

Example : $\phi = EX_{D,\mathcal{O}} \phi'$

Let x be a variable representing an event C be a set of variable names already in use $MSO(\phi,x,C) = \exists y,x \leq_{\mathcal{O}} y \land MSO(\phi,y,C')$ with

- y is a fresh variable name (w.r.t. C and to the set $C_{x \leq_{\mathcal{O}} y}$ of variables used to encode $x \leq_{\mathcal{O}} y$)
- $C' = C \cup \{y\} \cup C_{x <_{\mathcal{O}} y}$;

From Hypol to MSO

Immediate corollaries:

Corollary

Hypol $\setminus EX_{\equiv,\mathcal{O}_i}$ is decidable for safe Petri nets

<u>Proof</u>: Equivalence edges are not used. MSO decidable for graph grammars [*Courcelle*90]

Checking $\mathcal{N} \models \phi$ = checking $\mathcal{G}_{\mathcal{N}} \models \mathit{MSO}(\phi)$

Corollary

MSO is undecidable on execution graphs of safe Petri nets

<u>Proof</u>: Consistent with former theorems. Further $G_{\mathcal{U}_{\mathcal{N}}}$ may contain infinite grids minors (a condition for undecidability of MSO [*Robertson*&Seymour91])

Observable nets & Layeredness

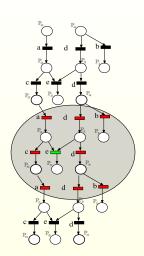
Distance between events

dist(e,f) = maximal number of edges between $\{e,f\}$ and their common past

Balls

The K - Ball of e in $\mathcal{U}_{\mathcal{N}}$ is the set

$$Ball_K(e) = \{ f \in E \mid dist(e, f) \le K \}$$

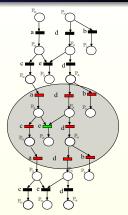


Equivalence decision on K-layered graphs

Definition: K-layeredness

 \mathcal{N} is K-layered for observations $\mathcal{O}_1, \dots, \mathcal{O}_a$ iff :

- the K-ball of every event e of $\mathcal{U}_{\mathcal{N}}$ is finite
- $\forall \mathcal{O}_i, dist(e, f) > K$ implies $e \not\equiv_i f$
- $e \equiv_i f$ can be decided from the contents of $Ball_K(e)$ and $Ball_K(f)$



Proposition

Let \mathcal{N} be a K-layered safe Petri net (w.r.t. $\mathcal{O}_1, \ldots, \mathcal{O}_g$). Then, one can effectively compute a graph grammar $\mathcal{G}_{K,\mathcal{N}}$ that recognizes $G_{\mathcal{U}_{\mathcal{N}}}$

<u>Proof idea</u>: Hyperedges memorize K-balls of maximal events.

Equivalence decision on K-layered graphs

Corollary

Hypol model checking is decidable for K-layered nets.

Open question

Is K-layeredness decidable?

Theorem

Observable nets are K-layered for some $K \leq \max(2 \cdot k_c, 3 \cdot |T|)$

Corollary

Hypol model checking is decidable for Observable nets

Conclusion

Contributions

- A new partial order hyperlogic : Hypol
- Hypol Model checking decidable for *K*-Layered nets.
- A decidable subclass : Observable nets

Open questions

- Complexity?
- Decidability of K-layeredness?
- Unbounded Petri nets?
- Other types of regular models?

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