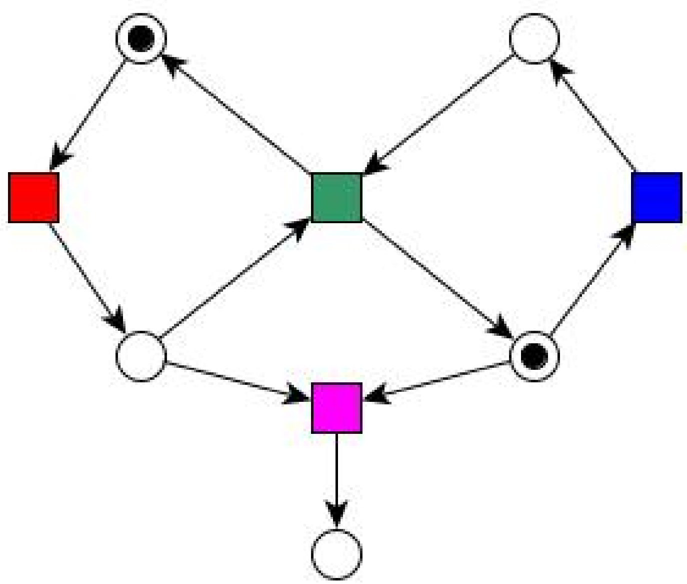
The Synthesis Problem for Regional Algebras

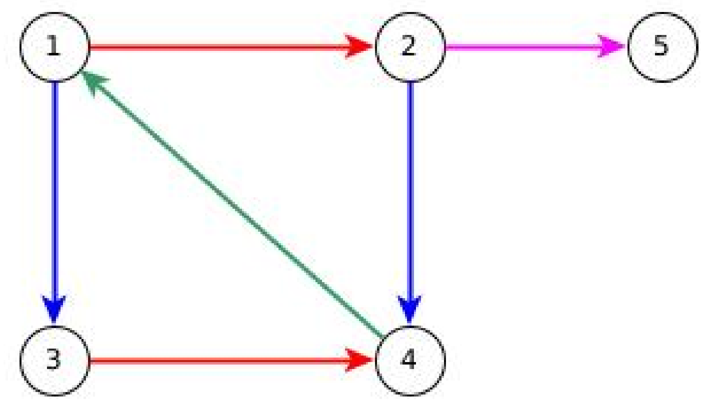
Luca Bernardinello, Carlo Ferigato, Lucia Pomello, Adrián Puerto Aubel
Università degli studi di Milano–Bicocca, JRC Ispra

24 April 2024

Elementary Net Systems

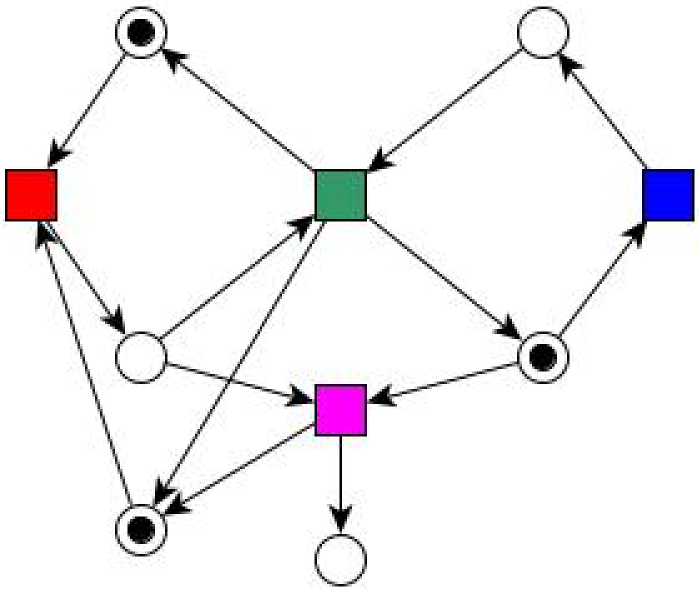
$\Sigma = (P, T, F, m_0) \qquad F \subseteq (P \times T) \cup (T \times P)$



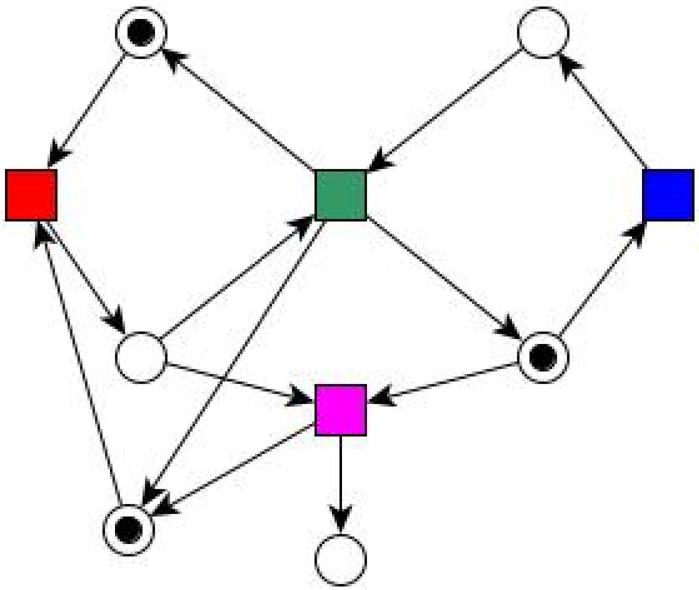


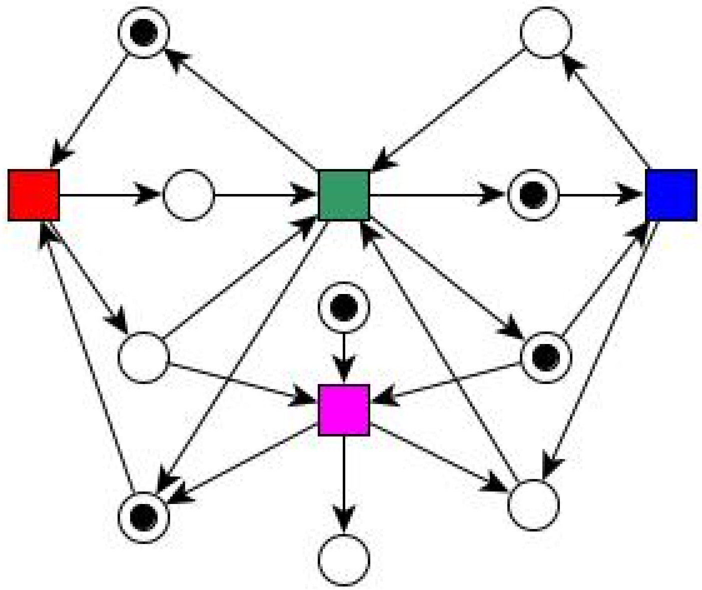
Marking graph

Implicit places



Place saturated net systems



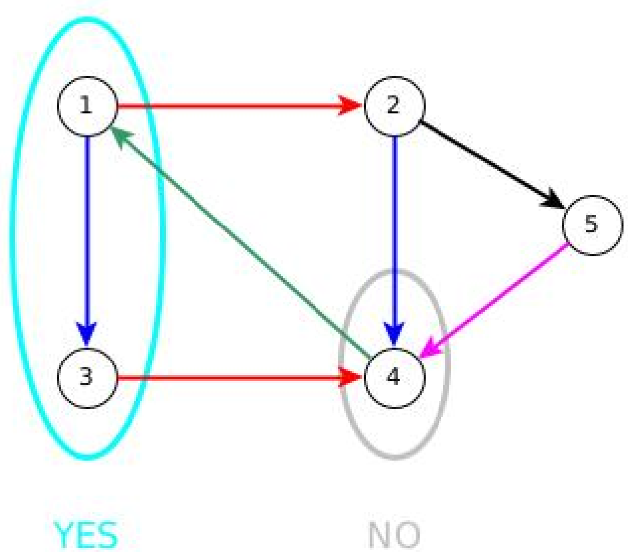


The synthesis problem

Given a labelled transition system $A = (Q, T, L)$, find an elementary net system $\Sigma = (P, T, F, m_0)$ such that its marking graph is isomorphic to A

Region theory

Andrzej Ehrenfeucht and Grzegorz Rozenberg, Partial (Set) 2-Structures. Part II: State Spaces of Concurrent Systems, Acta Informatica, 27, 4, 1990



Region set of states with a uniform crossing relation with each color

The synthesis problem for Elementary Net Systems

The synthesis problem is solvable for $A = (Q, T, L)$ iff

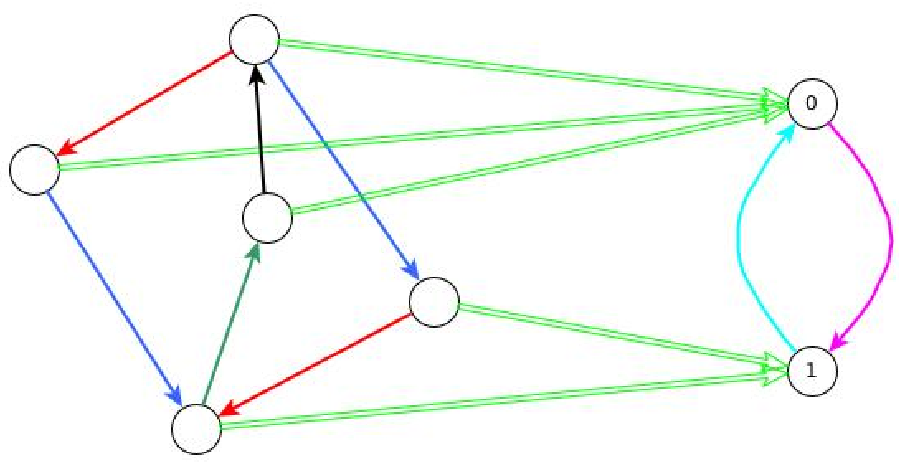
- regions of A separate Q
- regions of A prevent events in states where they are not enabled

Elementary (separated) transition systems

A general theory of regions

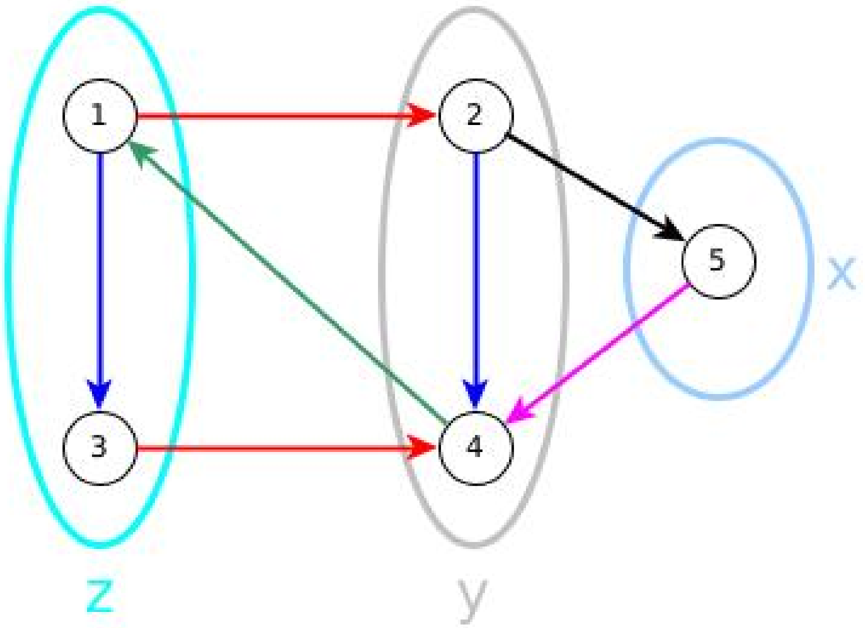
Éric Badouel and Philippe Darondeau, Theory of Regions, LNCS 1491, Springer, 1996

Types of nets: regions as morphisms



ENS type

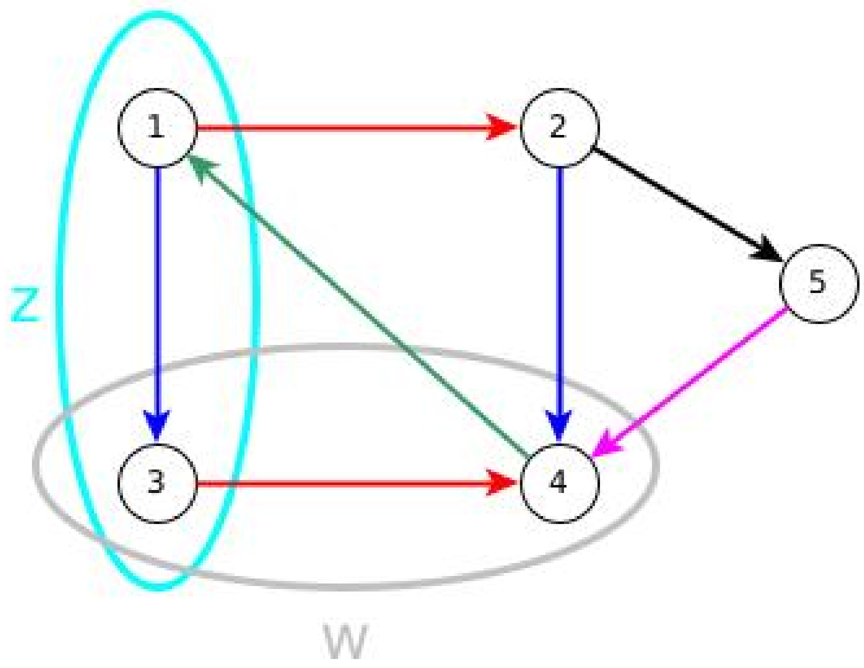
The algebraic structure of elementary regions



The set complement of a region is a region

The union of disjoint regions is a region

The algebraic structure of elementary regions



Incompatible regions

Duality state–property

A state is (described by) a set of properties

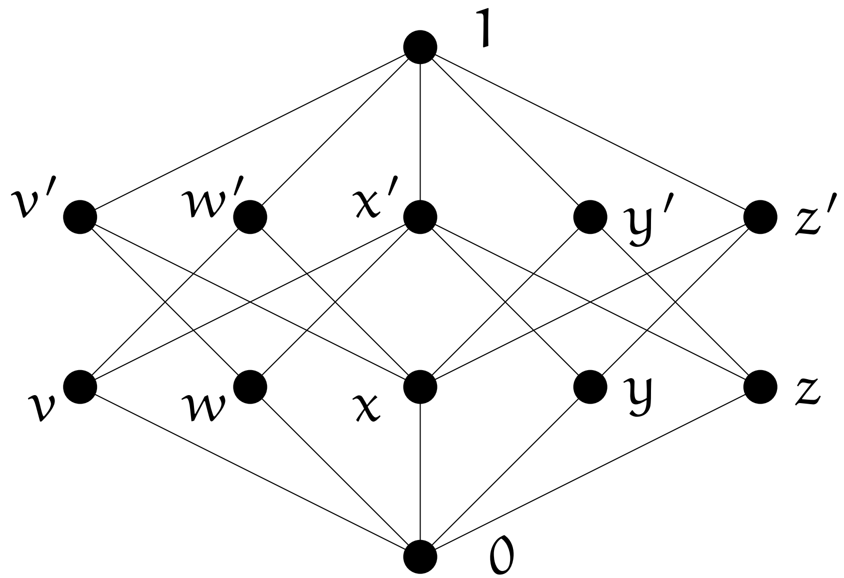
A property is a set of states

"Classical" approach: Boolean algebra of properties

Distributed system: ? [finite speed of signals]

Not all subsets of states are regions

Regions as partially ordered sets



Orthomodular posets, quantum logics, partial Boolean algebras

Boolean subalgebras

$BA_1 = \{0, x, w, v, x', w', v', 1\}$

$BA_2 = \{0, x, y, z, x', y', z', 1\}$

Concrete orthomodular posets

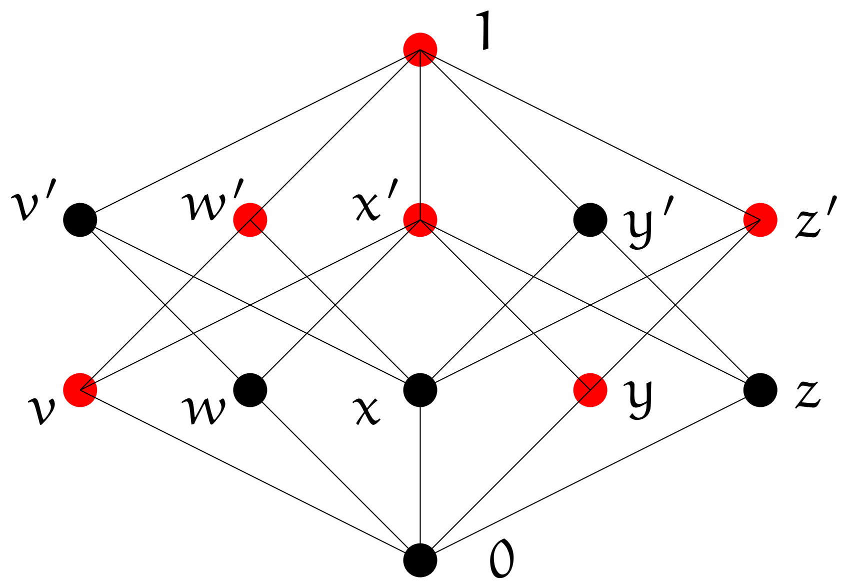
- Elements are subsets of a given set U; partial order is set inclusion; states "separate" elements

$G = \{1, \ldots, 6\}$

$\mathrm{Even}_6 = \{H \subseteq G \mid \mathrm{card}(H) \text{is even}\}$

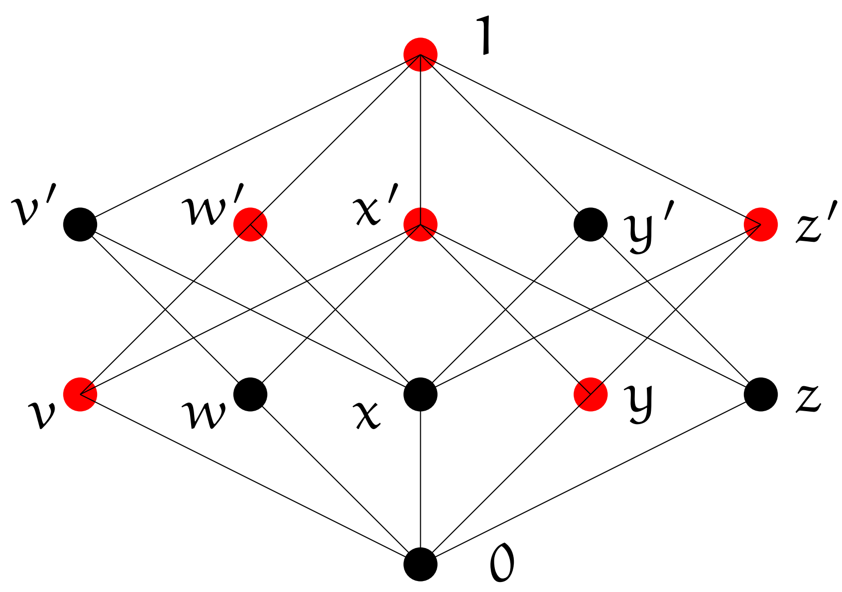
- Regional algebras are concrete, regular

States of orthomodular posets



A state is a subset of elements such that its projection on each Boolean subalgebra is a maximal filter

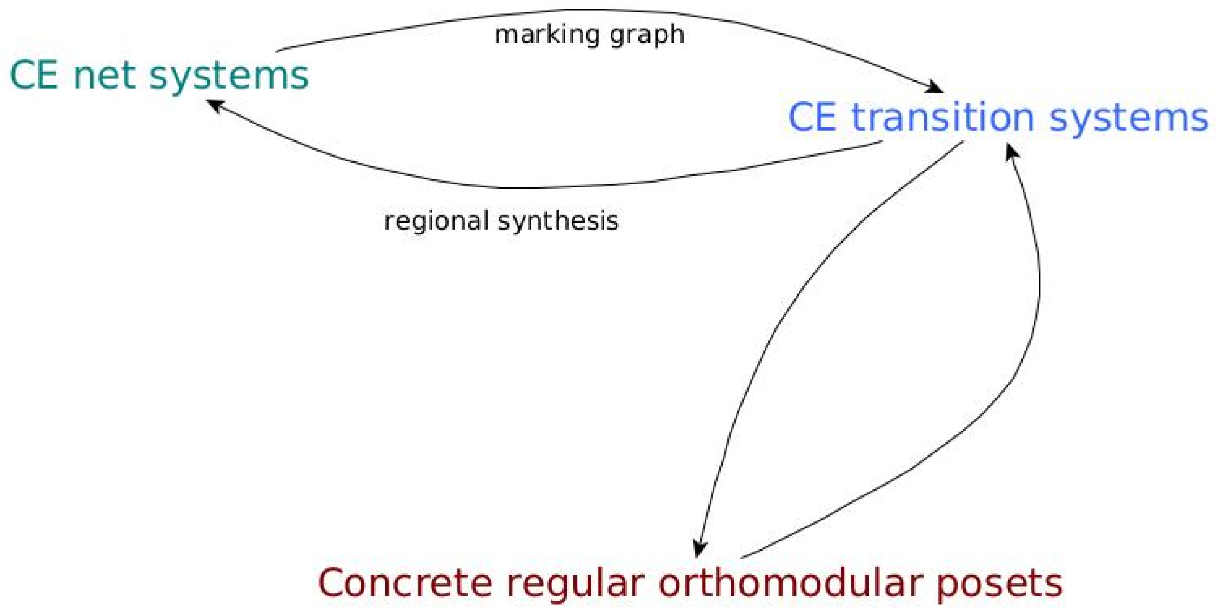
States of orthomodular posets



A state is a subset of elements such that its projection on each Boolean subalgebra is a maximal filter (example: $s = \{1, w', x', z', v, y\}$)

Transition labels are ordered symmetric differences: $\langle s_1 \setminus s_2, \quad s_2 \setminus s_1 \rangle$

Regions and categories

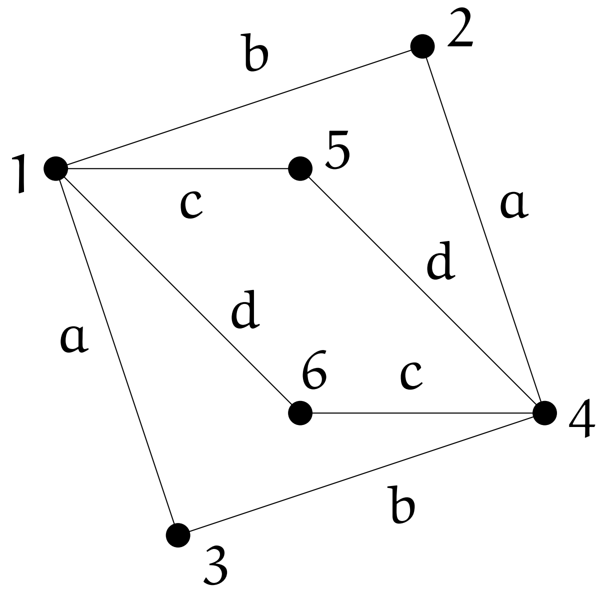


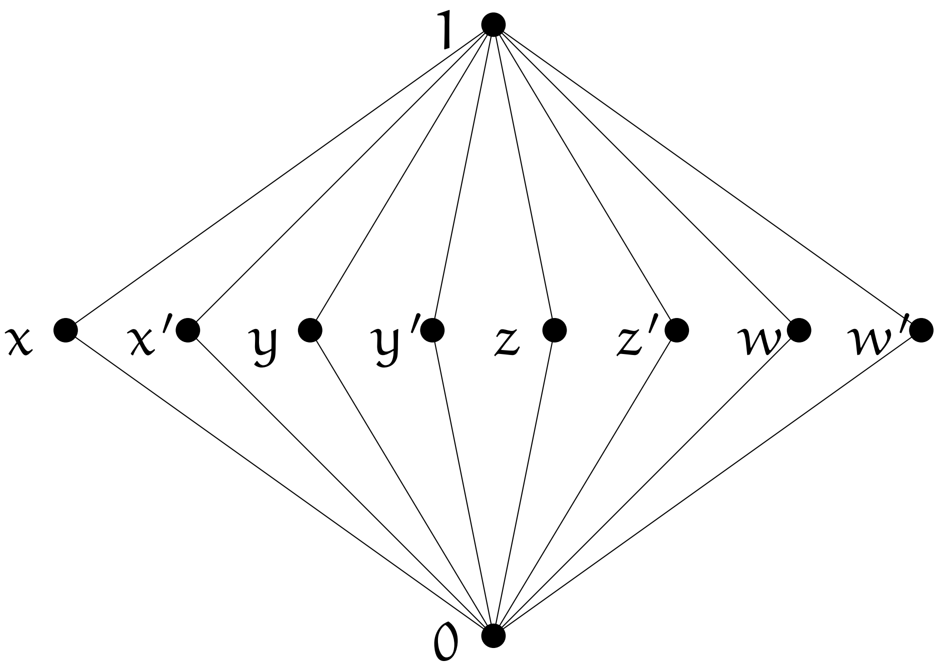
Characterization of regional orthomodular posets

- Triple intersection property
- Independent events testify incompatibility

Sufficient and necessary conditions?

States of orthomodular posets





Regions: $x = \{1, 2, 5\}, \quad x' = \{3, 4, 6\}, \quad y = \{1, 2, 6\}, \ldots$

Open problems

- Characterization of regional orthomodular posets
- Stability of regional orthomodular posets
- Limits in the category of regional orthomodular posets