RaPS, EPITA Rennes, April 24

Domains and event structures for fusions [categories not included]

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what is in a concurrent system?

- Process calculi view: systems are terms of a (possibly) free algebra, and operators represent basic concurrency features (parallel, sequence, non-determinism…)
  - Of course, you need to give semantics to a process…
  - …and it would be better to be a “concurrent” one!
    - Also to exploit it for verification and the like.

a simple process, and its semantics

$$a.c \mid b$$

a simple process, and its semantics

$$a.c \mid b$$

operational

$$a.c \mid b \rightarrow_a c \mid b \rightarrow_b c \mid \mathbf{0} \rightarrow_c \mathbf{0} \mid \mathbf{0}$$

$$a.c \mid b \rightarrow_b a.c \mid \mathbf{0} \rightarrow_a c \mid \mathbf{0} \rightarrow_c \mathbf{0} \mid \mathbf{0}$$

$$\ldots$$

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$$\ldots$$

$$\{a.b.c, a.c.b, b.a.c\}$$ denotational

putting orders into the picture

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putting orders into the picture

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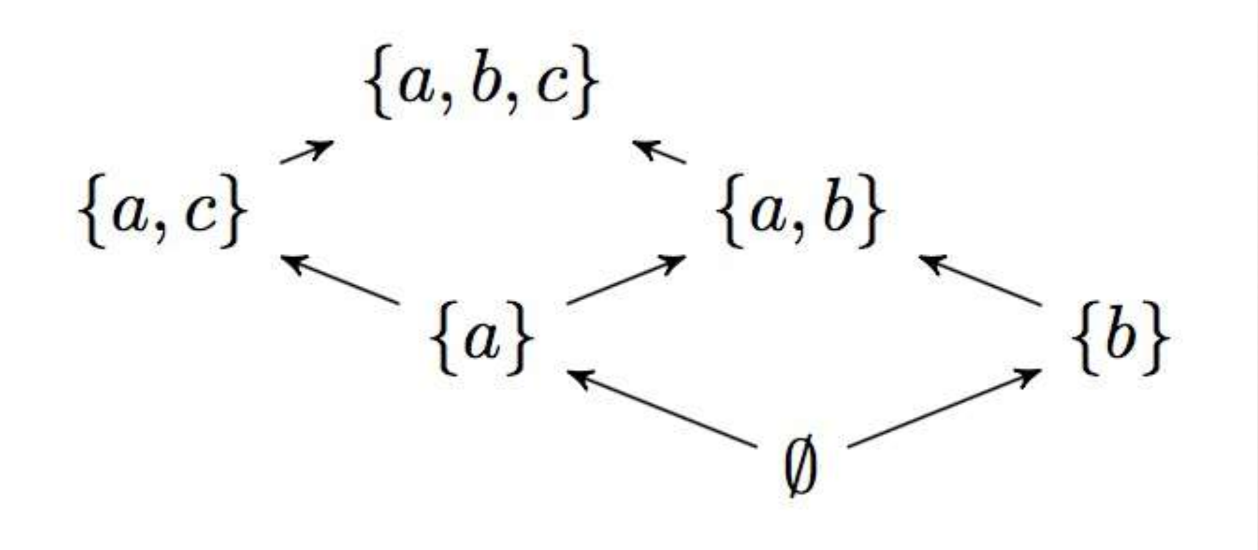
partial order of configurations



putting orders into the picture

$$a.c \mid b$$

$$\{a, b, c\}$$

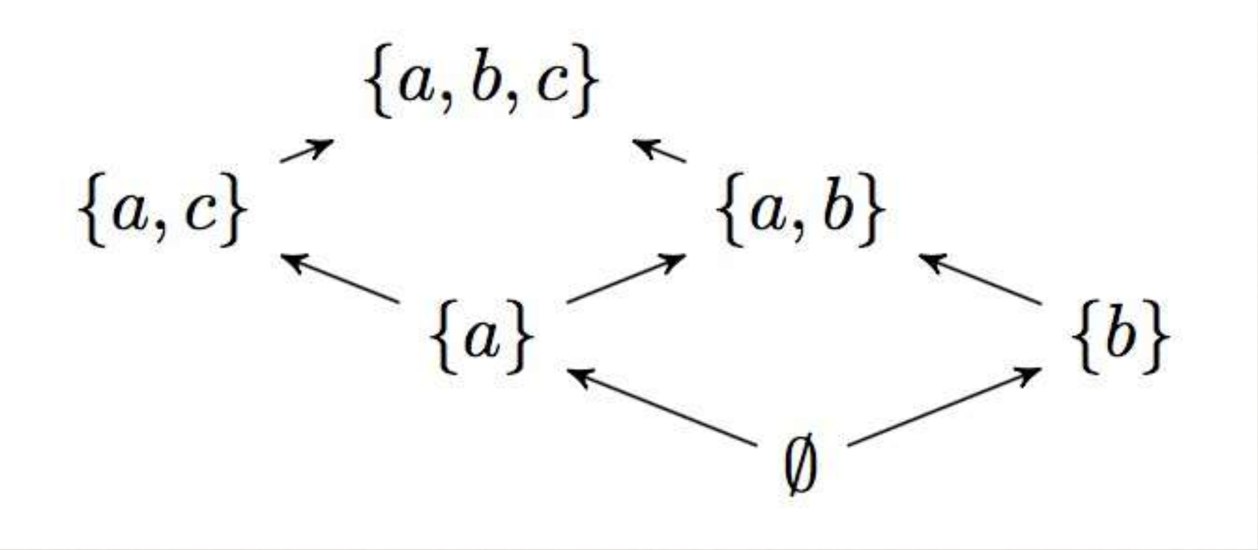
$$\emptyset \vdash a$$

$$\{a\} \vdash c$$

$$\emptyset \vdash b$$

entailment relation

partial order of configurations



event structures vs. configurations

$\langle E, \vdash, \# \rangle$

E a set of events

$\vdash \subseteq 2^E_f \times E$ an enabling relation

$\# \subseteq E \times E$ a conflict relation

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A *configuration* is a consistent C that can be linearised

$$\{e_1, \ldots, e_{k-1}\} \vdash e_k$$

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$$C \vdash_0 e \text{ if } C \vdash e, C' \subseteq C, \text{ and } C' \vdash e \text{ implies } C' = C$$

from ESs to configurations

ESs generate a (coherent etc.) PO of configurations
(wrt. set inclusion)

$\emptyset \vdash_0 a$

$\{a\} \vdash_0 c$

$\emptyset \vdash_0 b$

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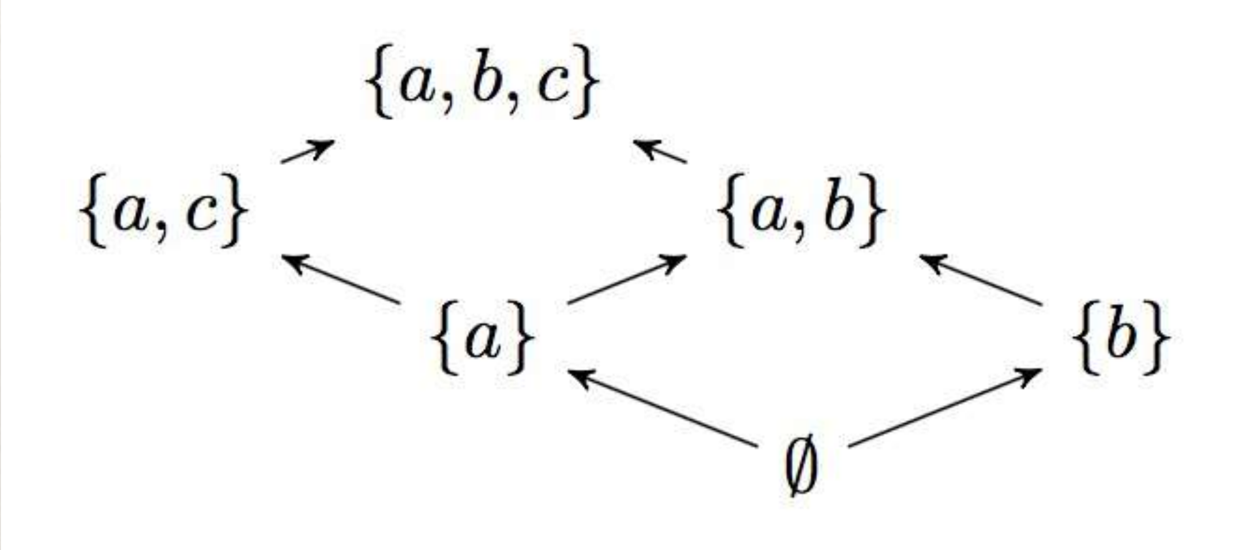

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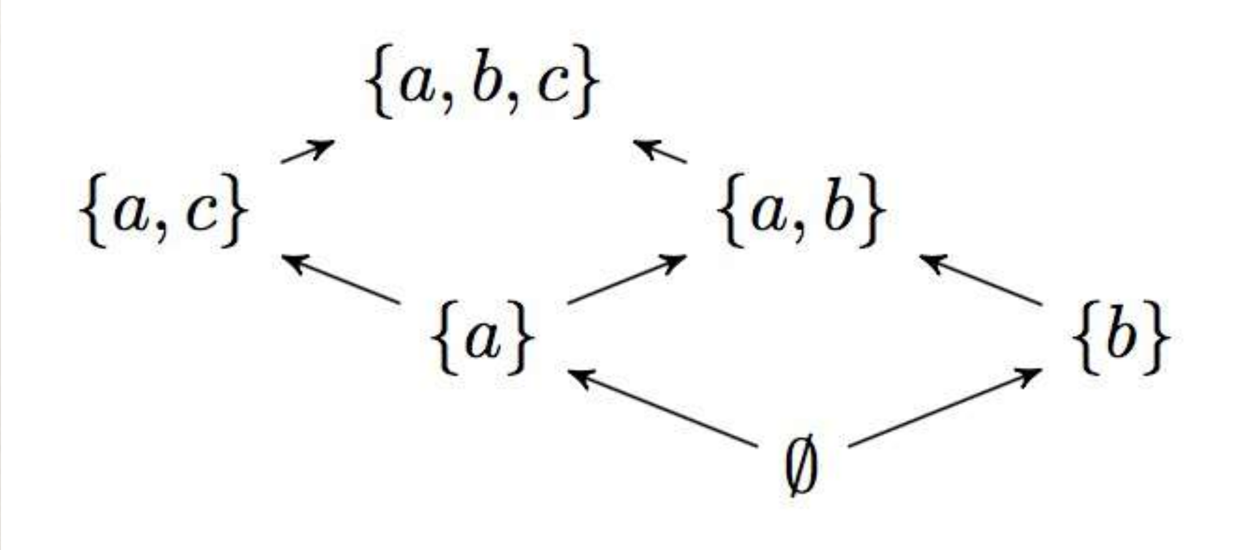

An ES is *prime* if $X \vdash e$ and $Y \vdash e$ imply $X \cap Y \vdash e$

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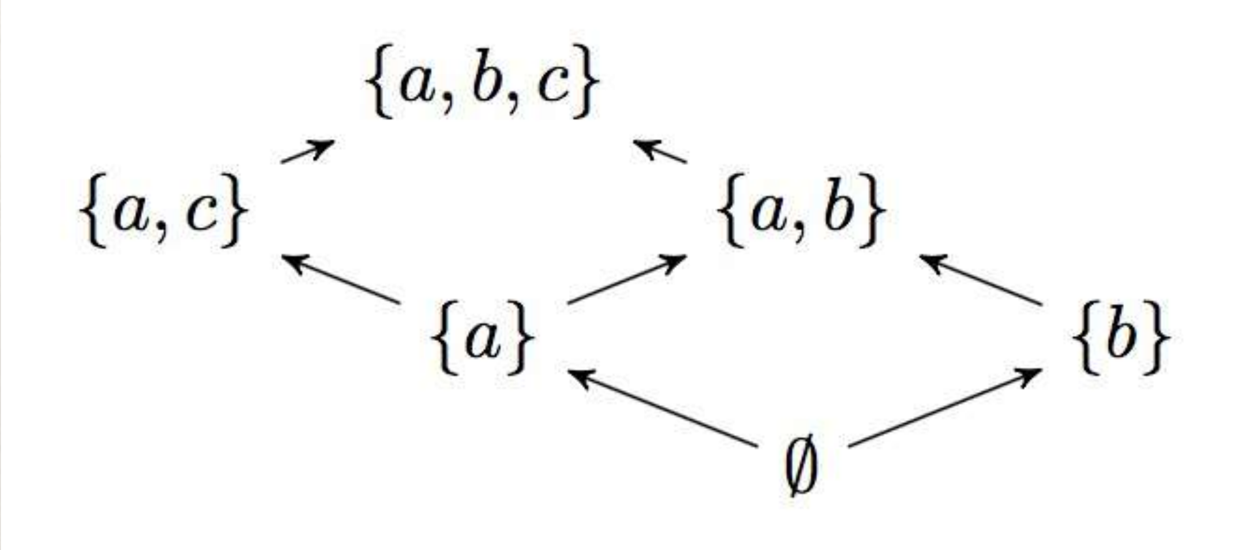

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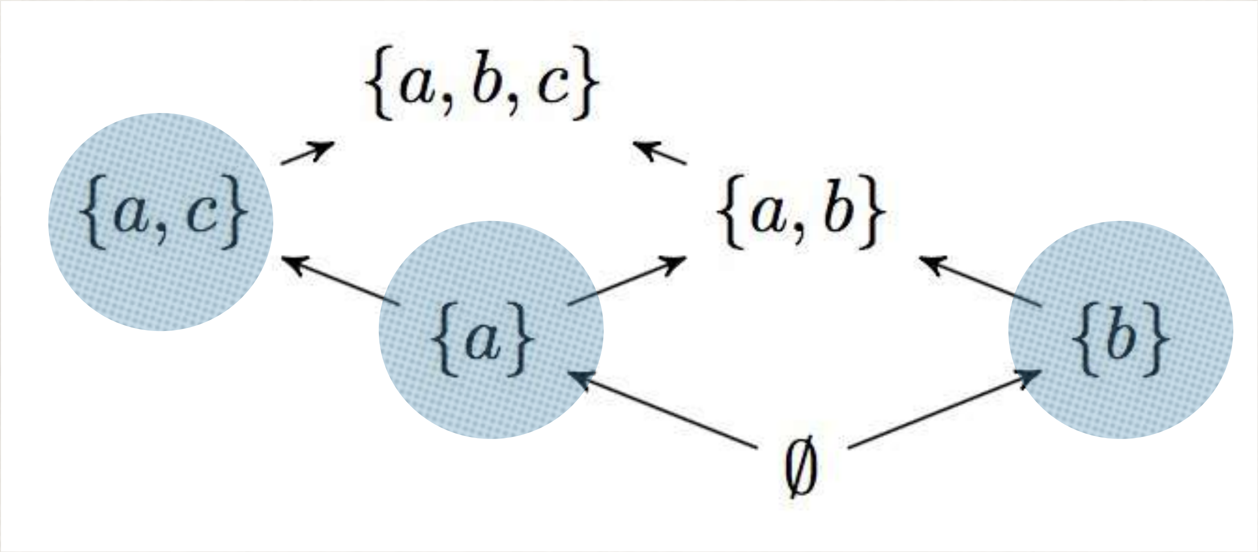
[each event has a minimal cause]

prime elements

An element p is prime if $p \sqsubseteq \bigsqcup X$ then $\exists x \in X . p \sqsubseteq x$

[a prime is a cause of any configuration it belongs to]



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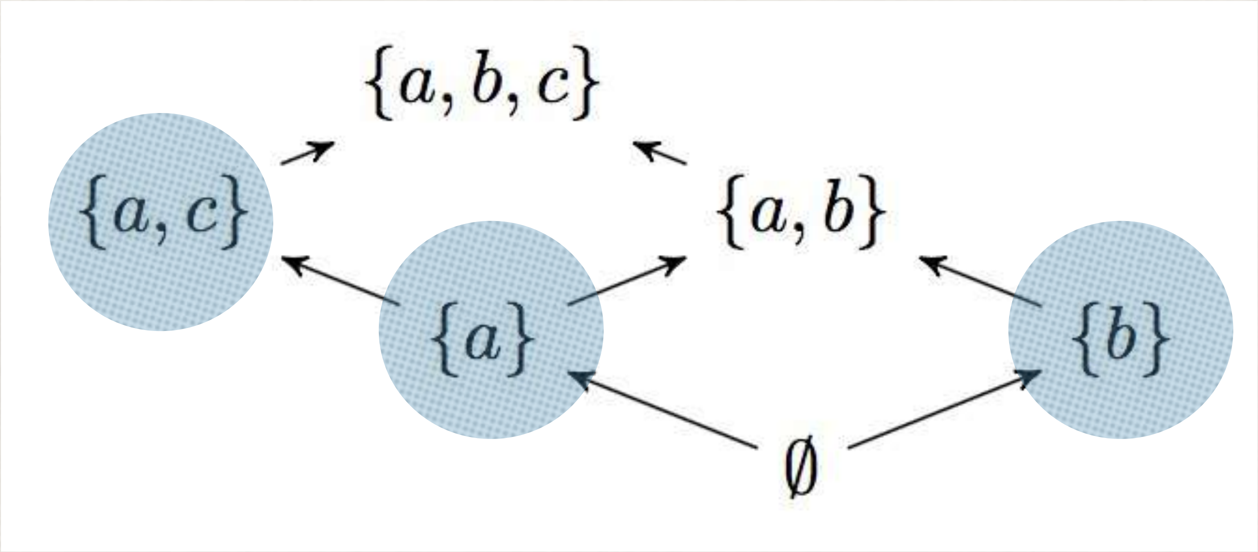

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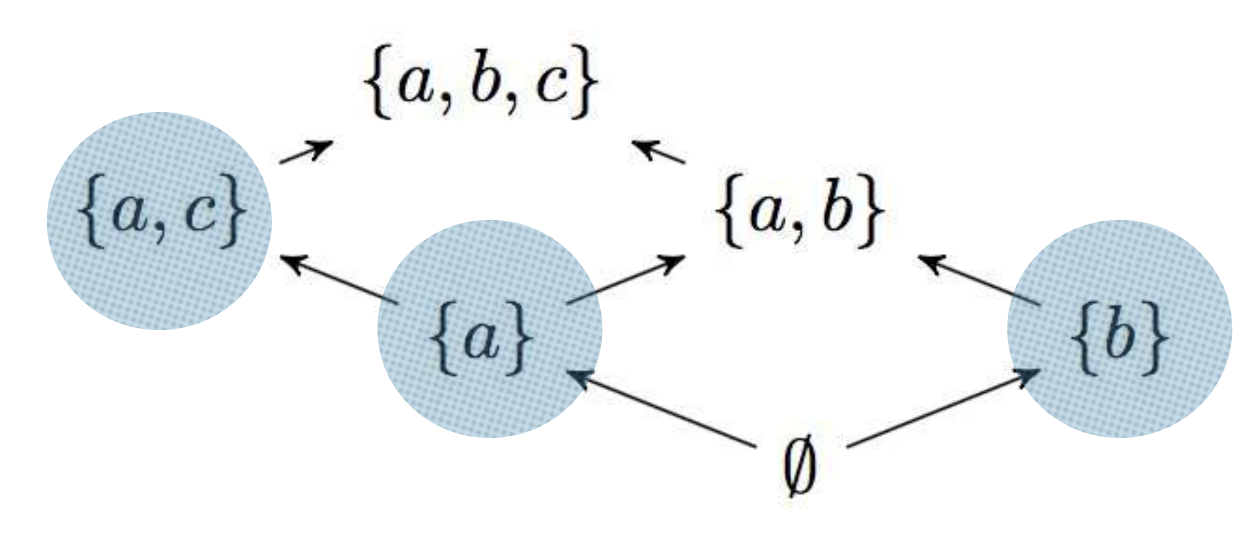

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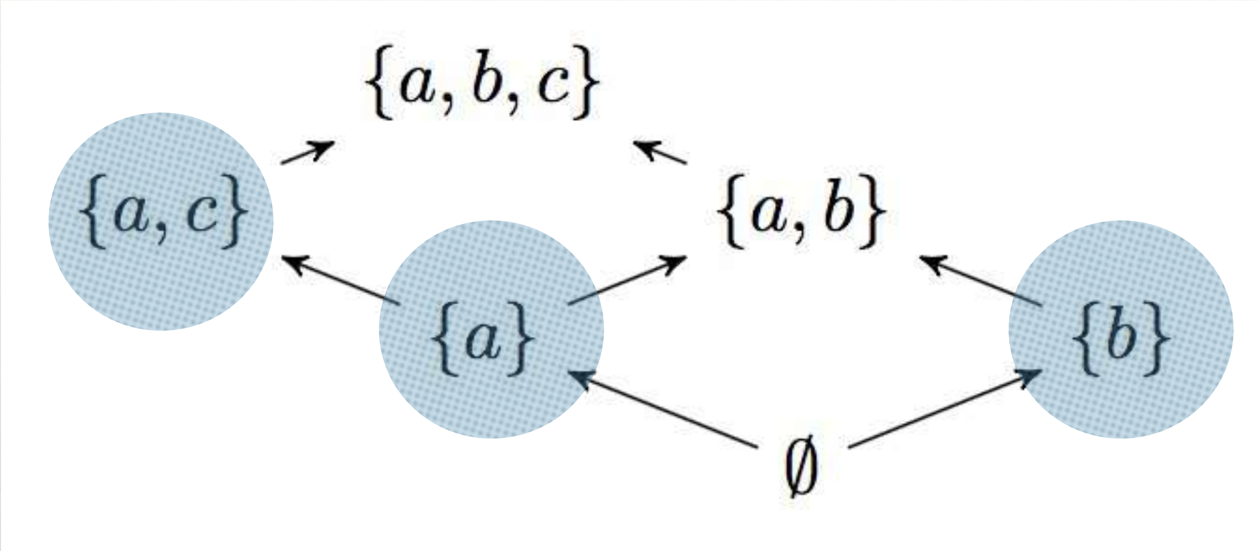
$$\forall d \in D.\, d = \bigsqcup (d \downarrow \cap\, pr(D))$$

from configurations to events

A prime PO generates a prime ES



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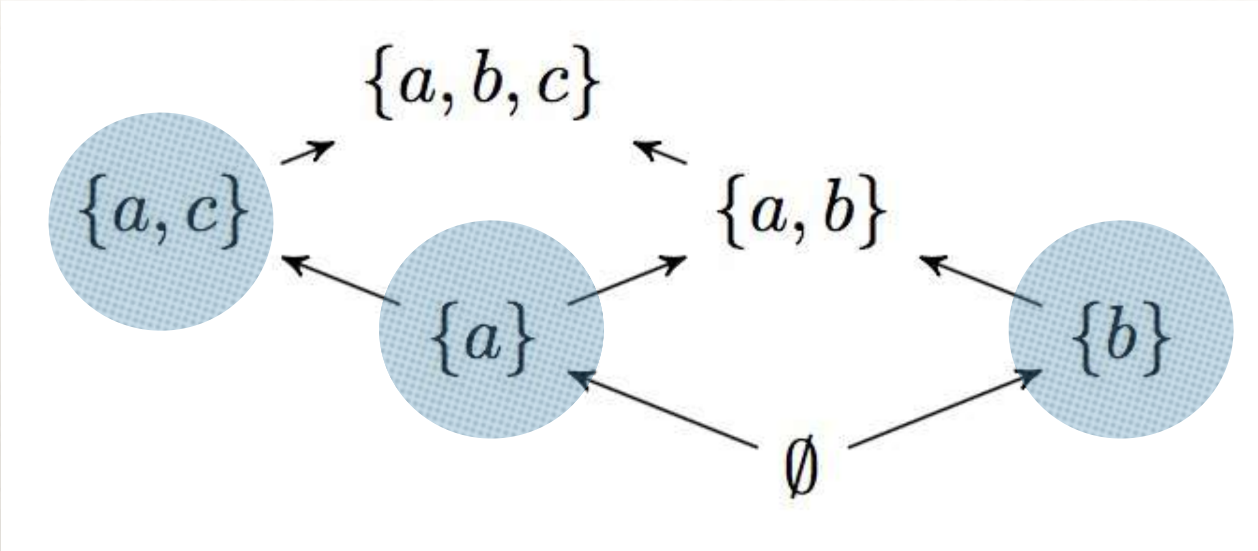

$$\{\{a\}, \{b\}, \{a, c\}\}$$

$$\emptyset \vdash_0 \{b\} \quad \emptyset \vdash_0 \{a\}$$

$$\{\{a\}\} \vdash_0 \{a, c\}$$

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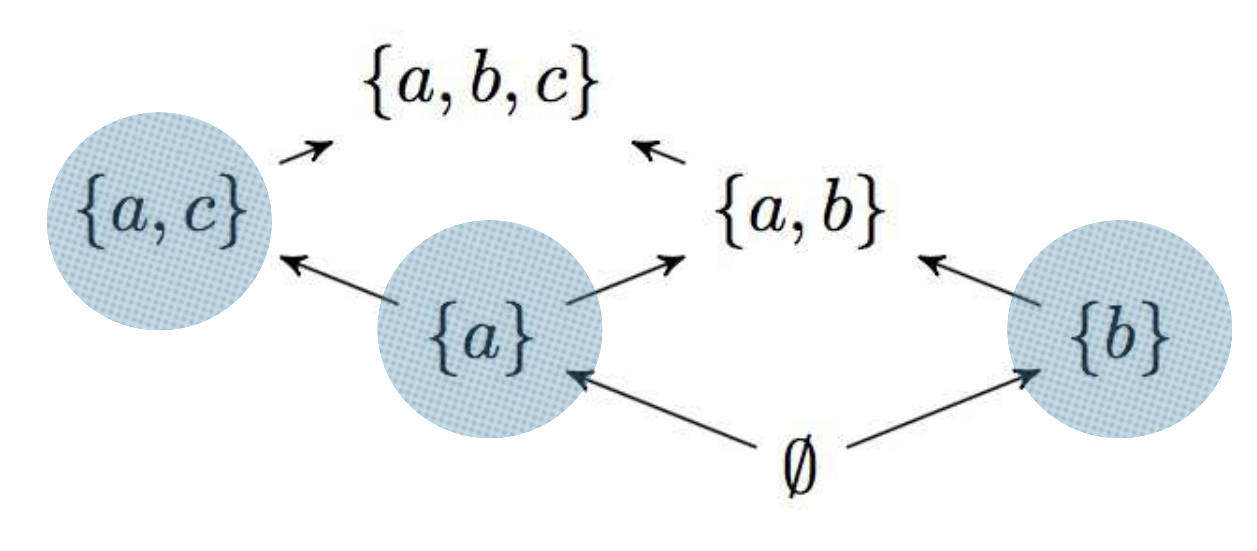

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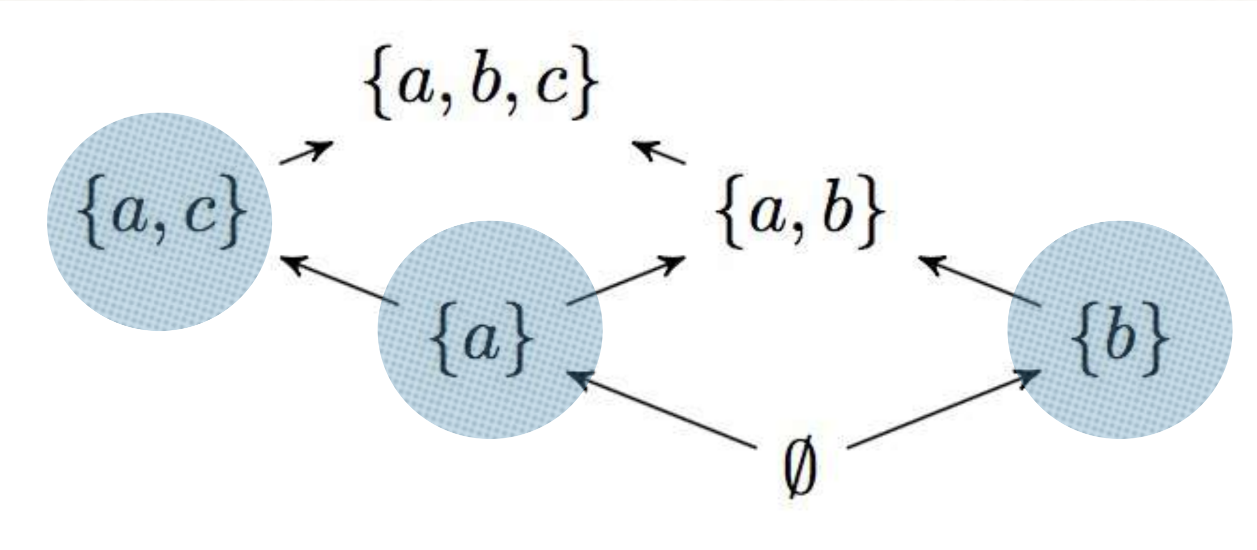

$$\{a, b, c\}$$

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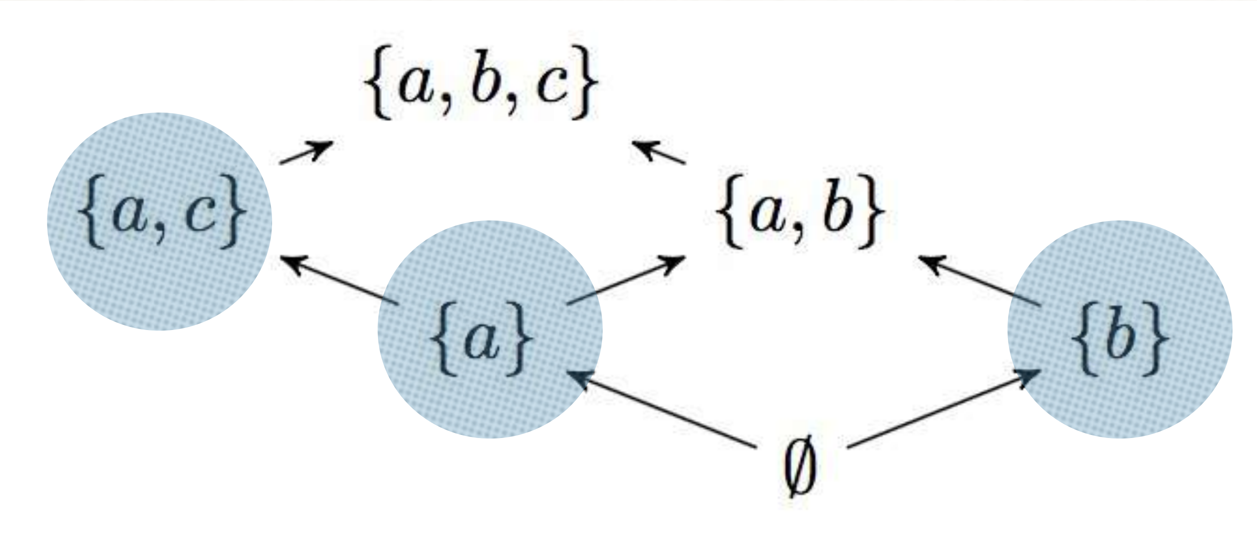

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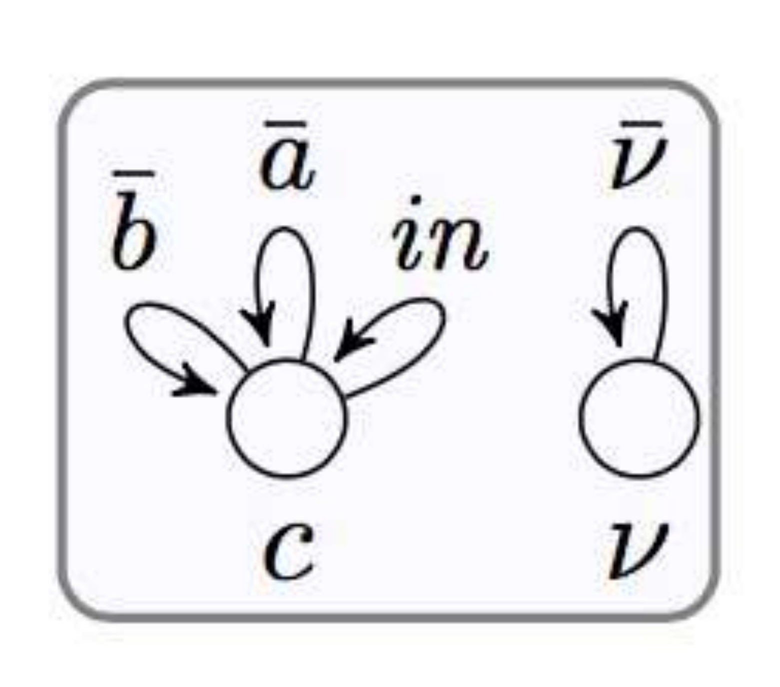
Moving back and forth between prime ESs and prime POs induces an isomorphism (actually, an equivalence of categories…)

a detour on graph rewriting



the initial graph

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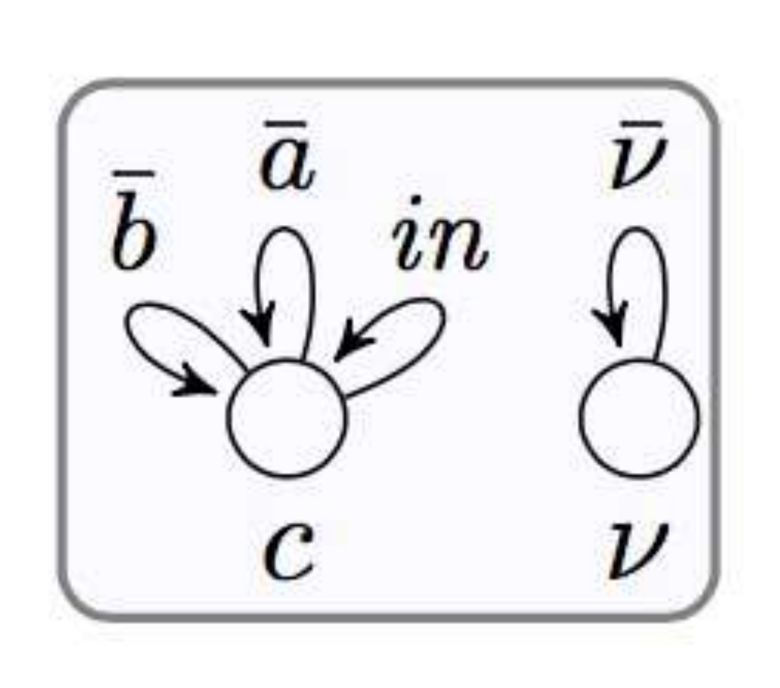
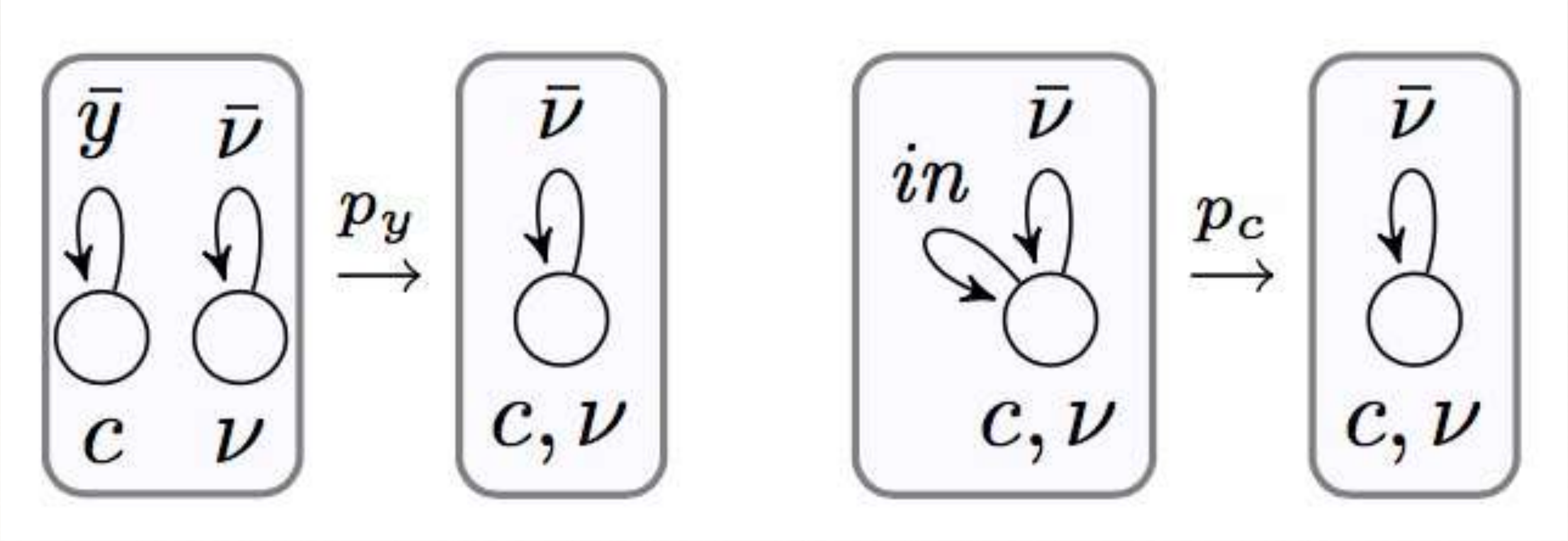
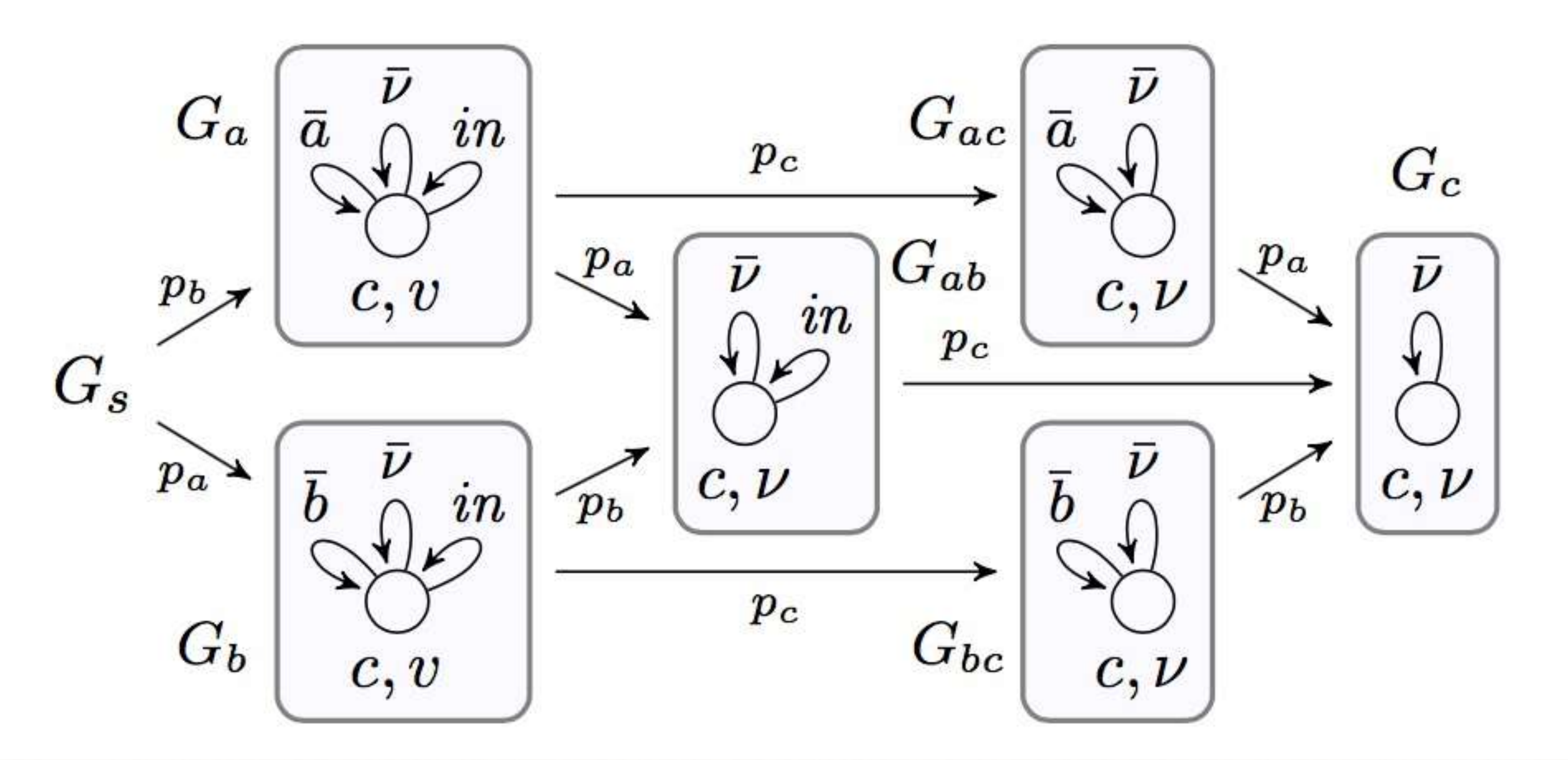


the initial graph

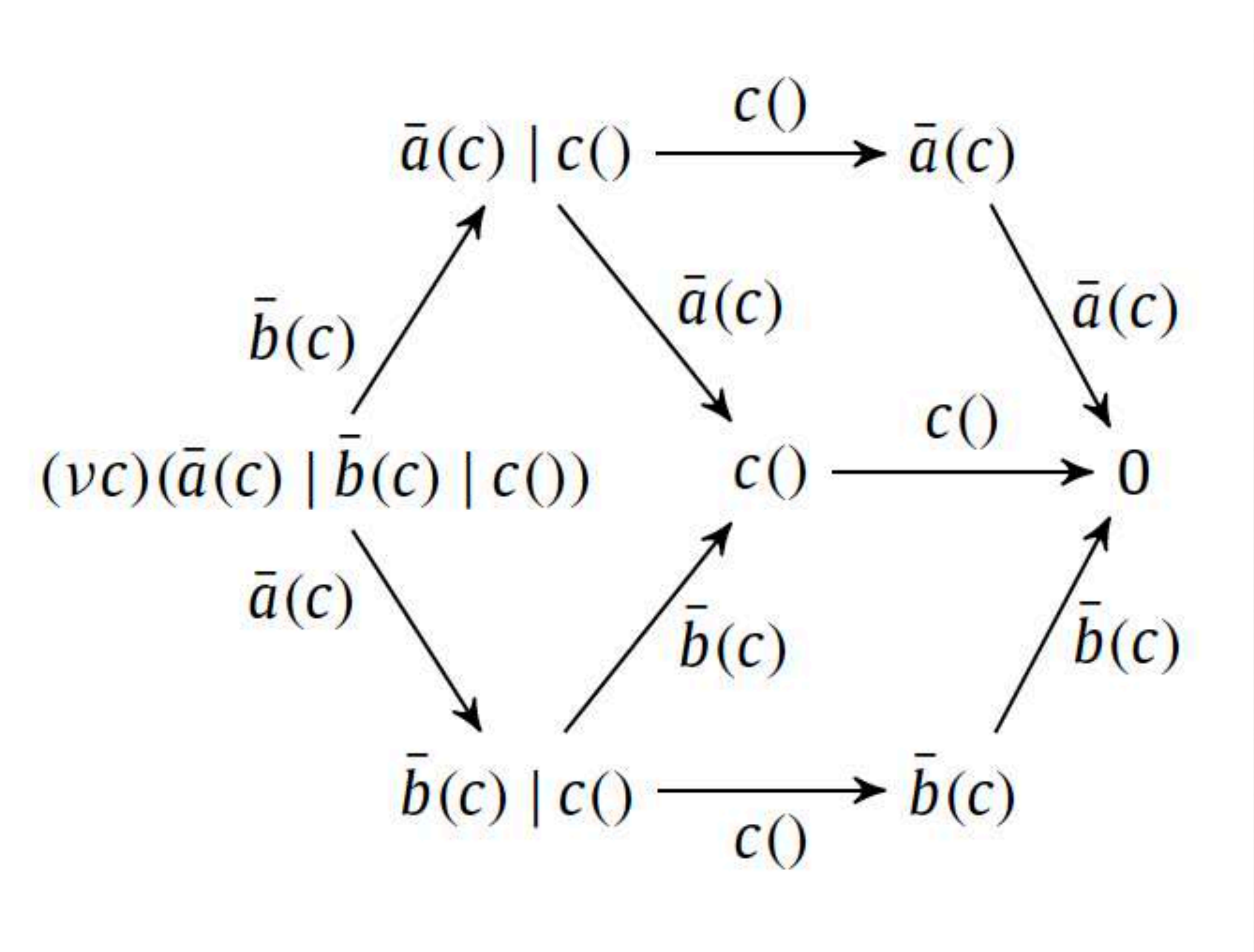
the rules



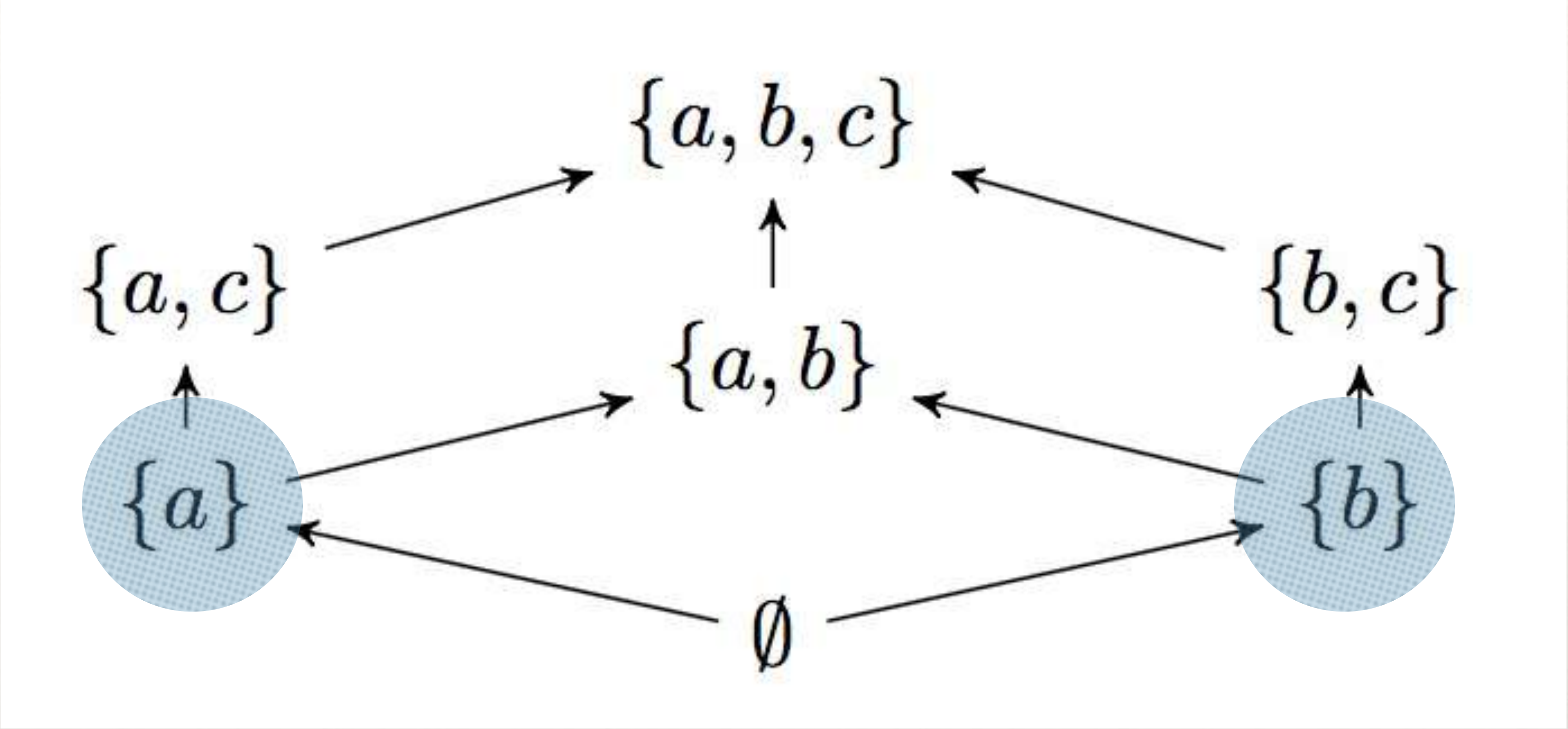
the semantics



going back to processes…

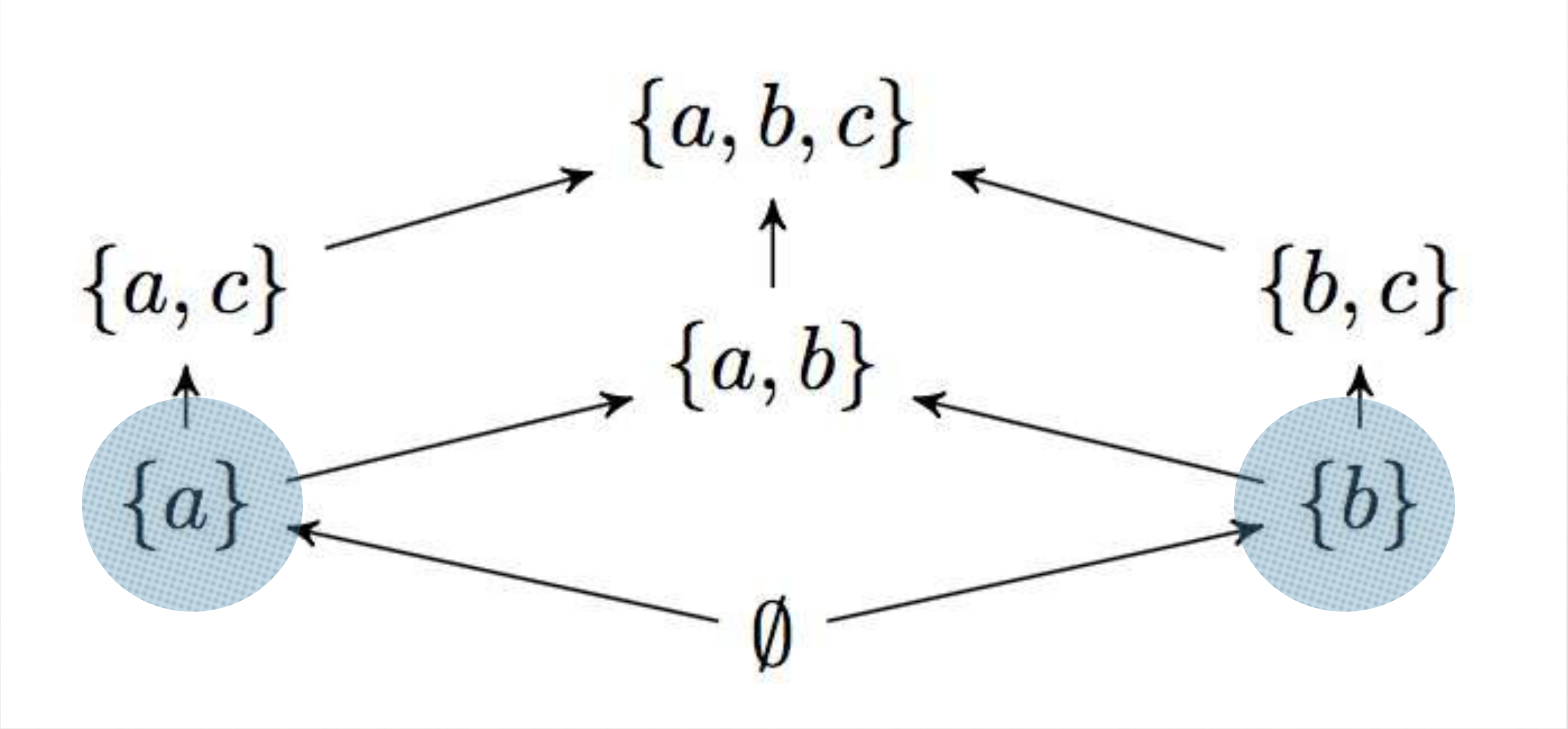


the PO of configurations



{a, c} and *{b, c}* are neither primes nor the sup of the primes they contain

the PO of configurations



{a, c} and *{b, c}* are neither primes nor the sup of the primes they contain

[either *a* or *b* suffices for *c*]

irreducible elements

An element *i* is irreducible if $i = \bigsqcup X$ then $i \in X$

irreducible elements

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[i is irreducible iff it has a unique predecessor $p(i)$]

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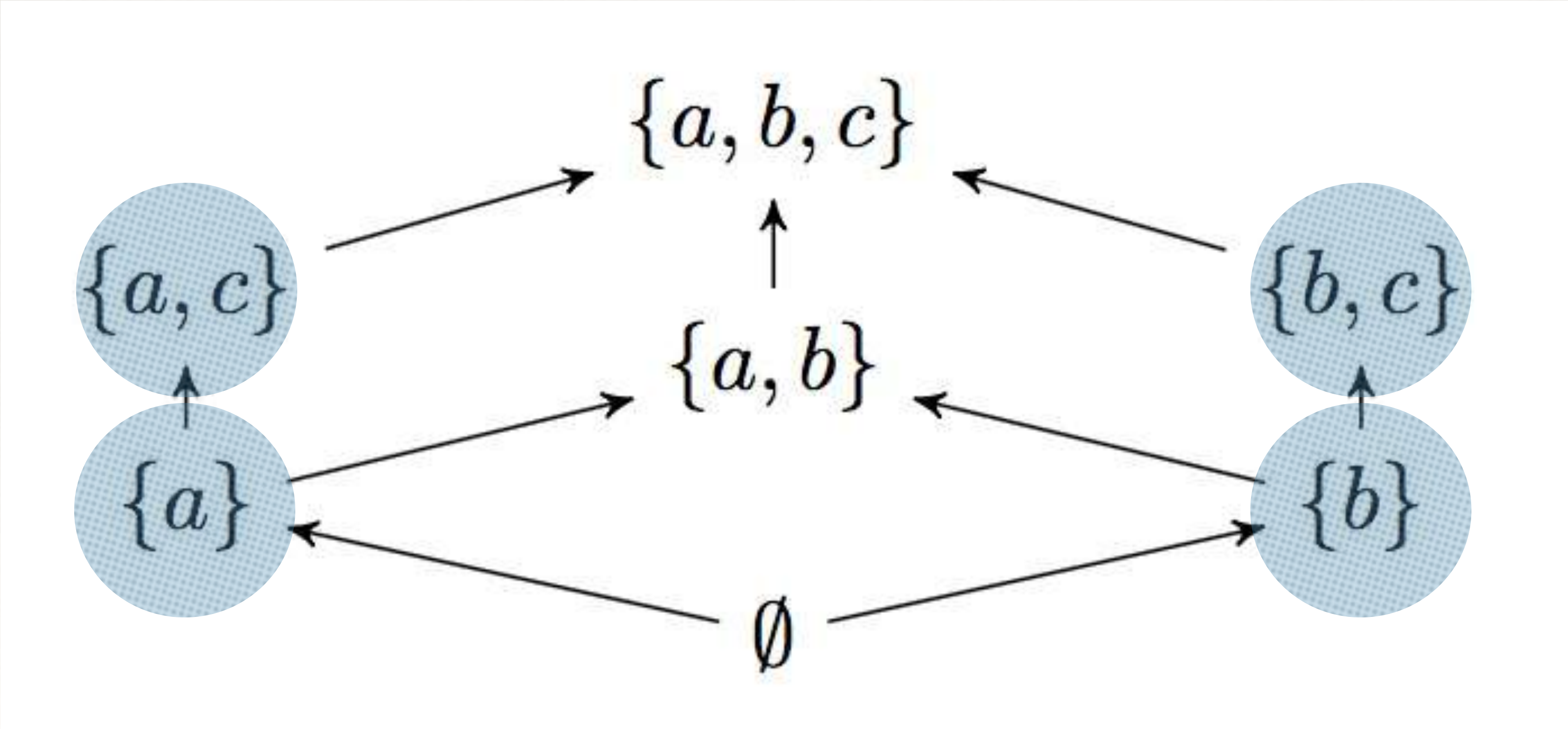
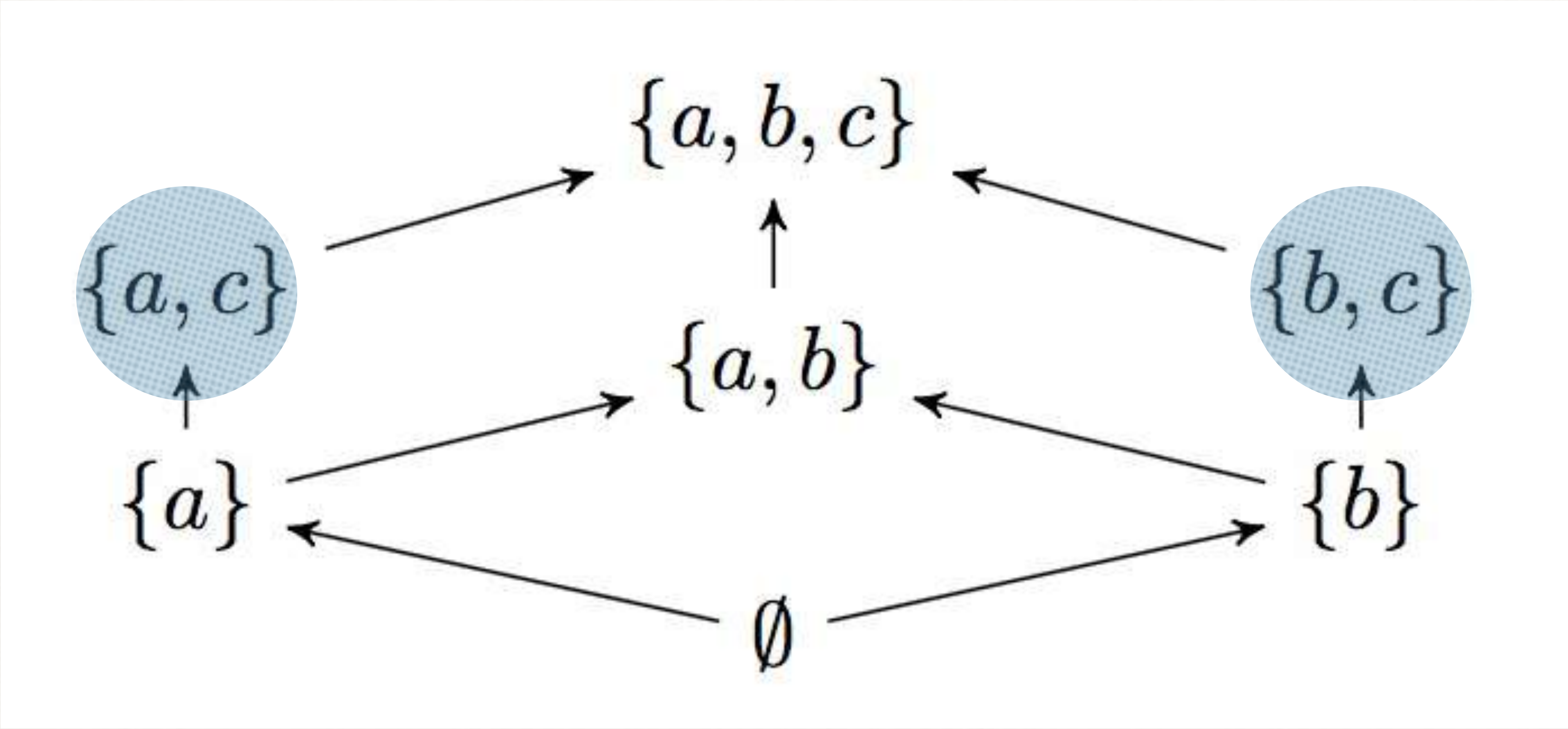
[*i* is irreducible iff it has a unique predecessor *p(i)*]



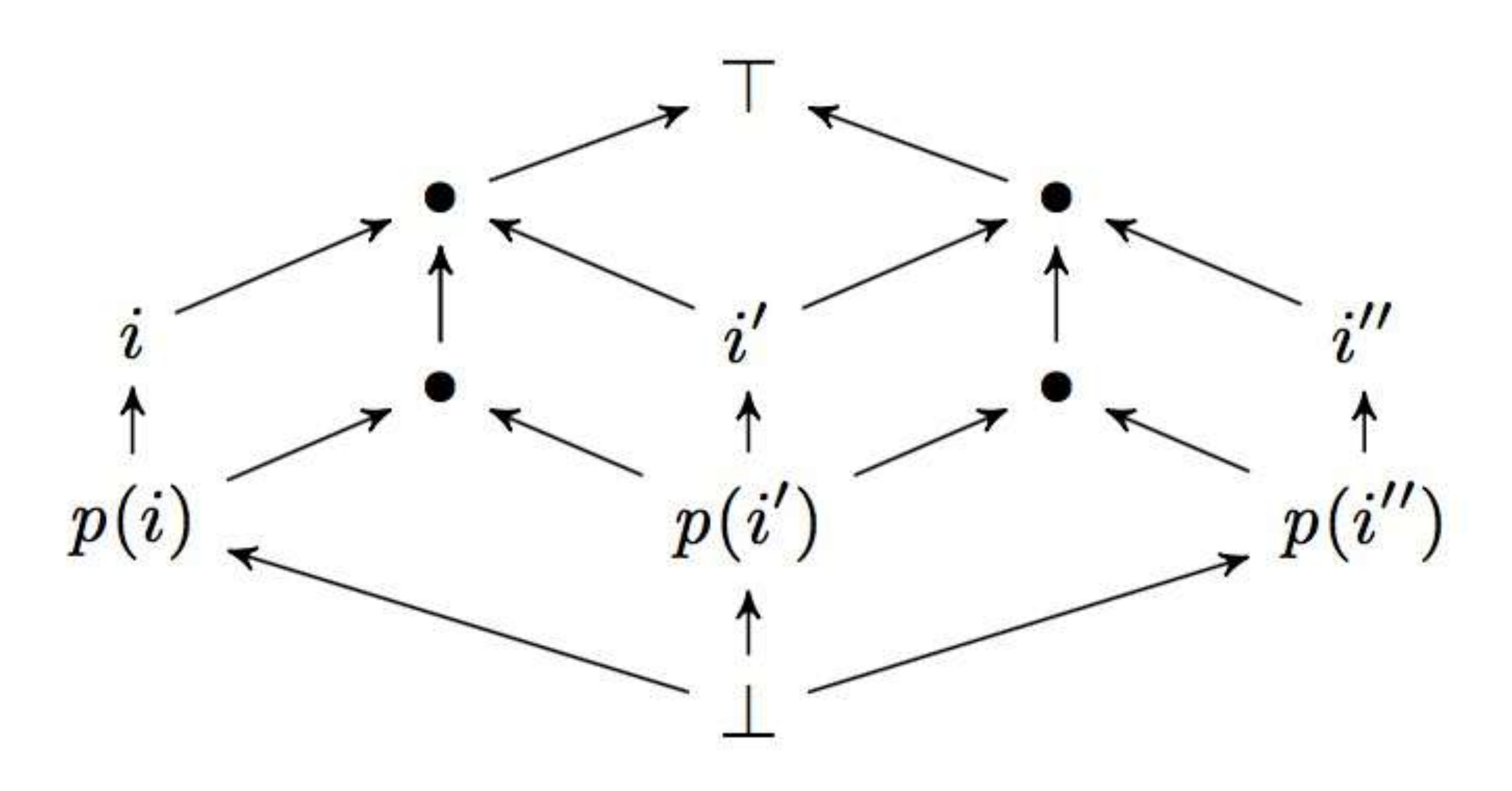
a relation on irreducibles

$$i \leftrightarrow i' \text{ if } i \sqcup p(i') = p(i) \sqcup i' \neq p(i) \sqcup p(i')$$



[they represent the same event (with different causes)]

interchange is not transitive



weak prime POs

An irreducible *i* is weak prime if

$$i \sqsubseteq \bigsqcup X \text{ then } \exists i'.(i \leftrightarrow i' \text{ and } \exists x \in X.i' \sqsubseteq x)$$

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[a weak prime is a cause of any configuration it belongs to, up-to interchange]

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[a weak prime is a cause of any configuration it belongs to, up-to interchange]

A PO is weak prime if each irreducible is weak prime and each element is the sup of the irreducible it contains [plus some stuff on the transitive closure of interchange]

connected ESs

$$C \stackrel{e}{\frown} C' \text{ if } C \vdash_0 e, C' \vdash_0 e, \text{ and } C \cup C' \cup \{e\} \text{ consistent}$$

An ES is connected if $C \vdash_0 e$ and $C' \vdash_0 e$ implies $C(\stackrel{e}{\frown})^* C'$

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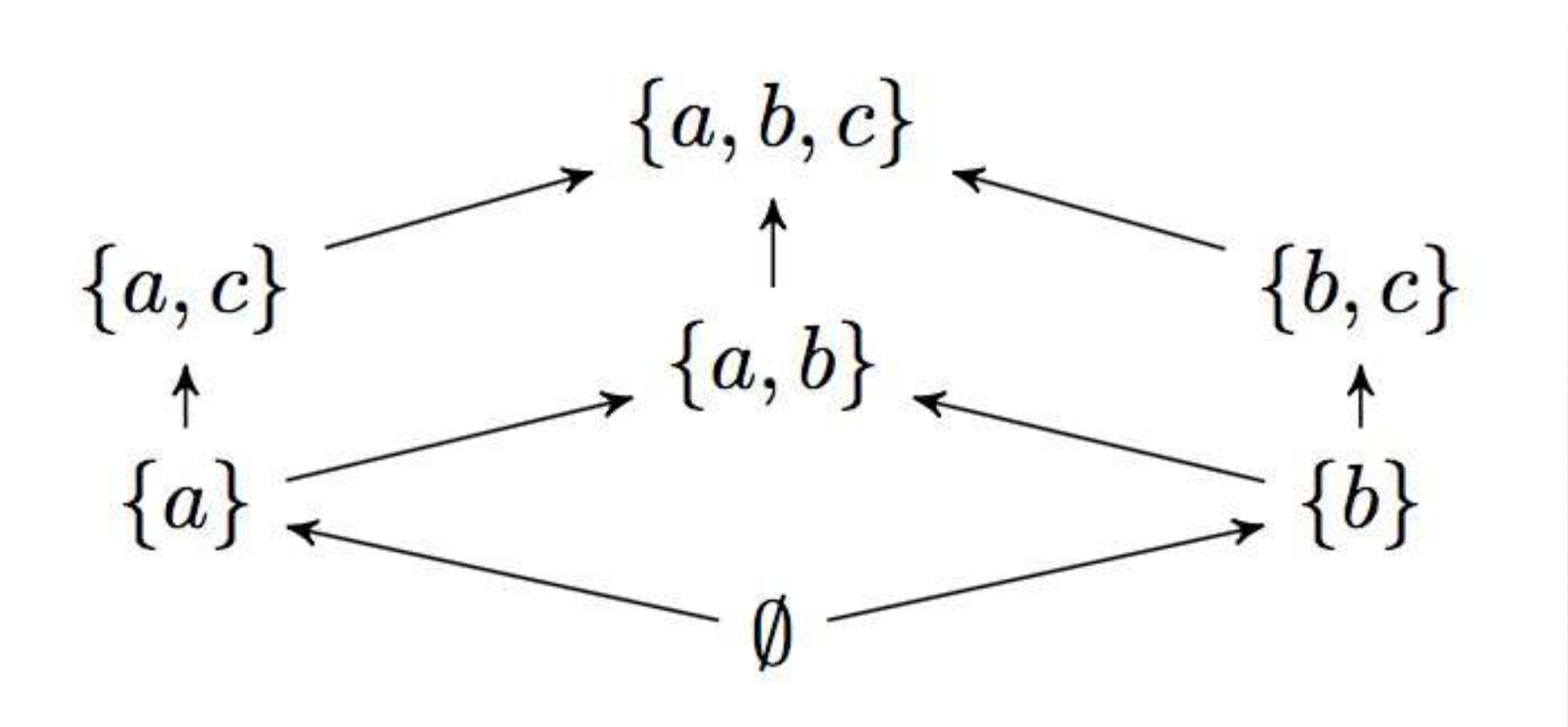
[the ES equivalent of PO interchange]

from configurations to events, weakly

A weak prime PO generates a connected ES



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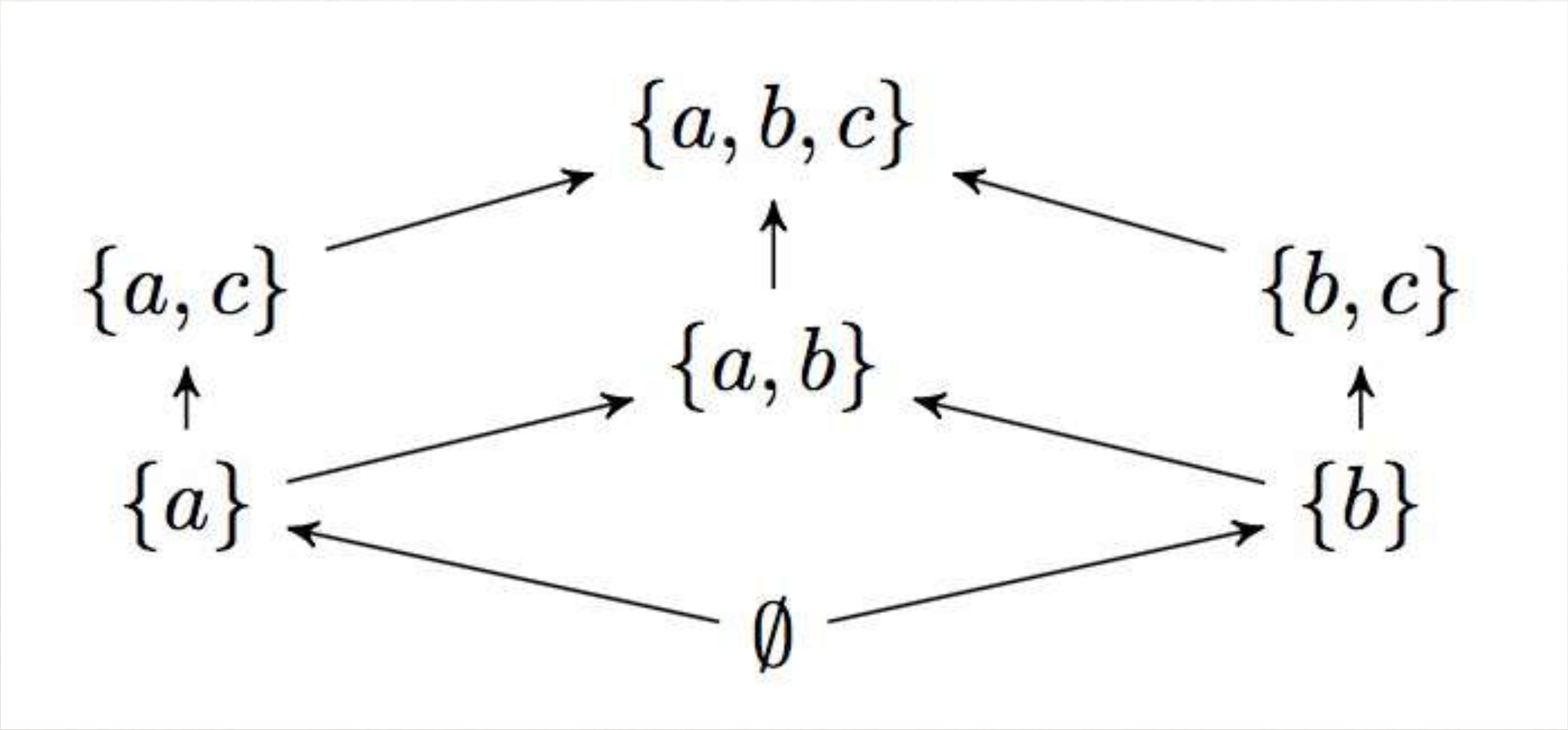

$$\{a, b, c\}$$

$$\{a\} \vdash_0 c$$

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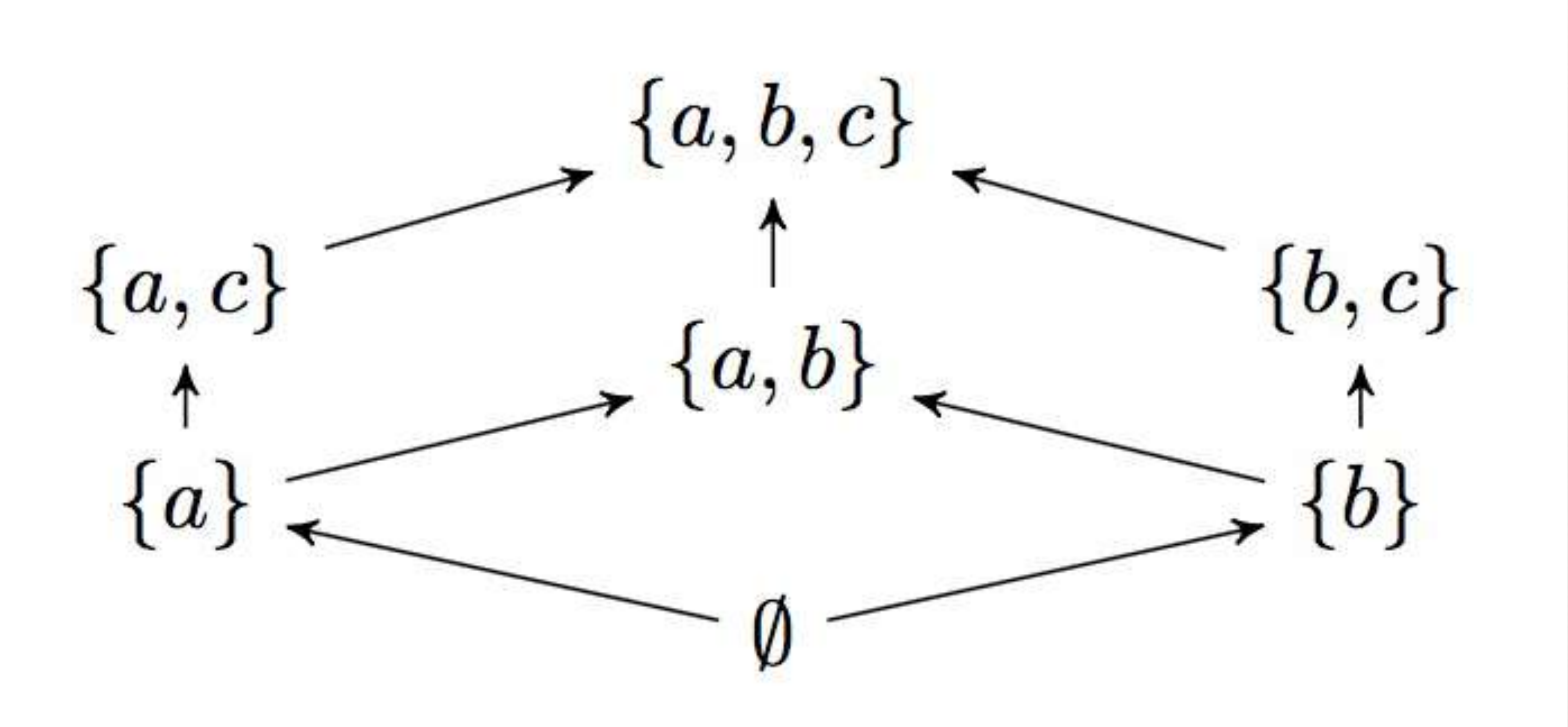

$\{a, b, c\}$

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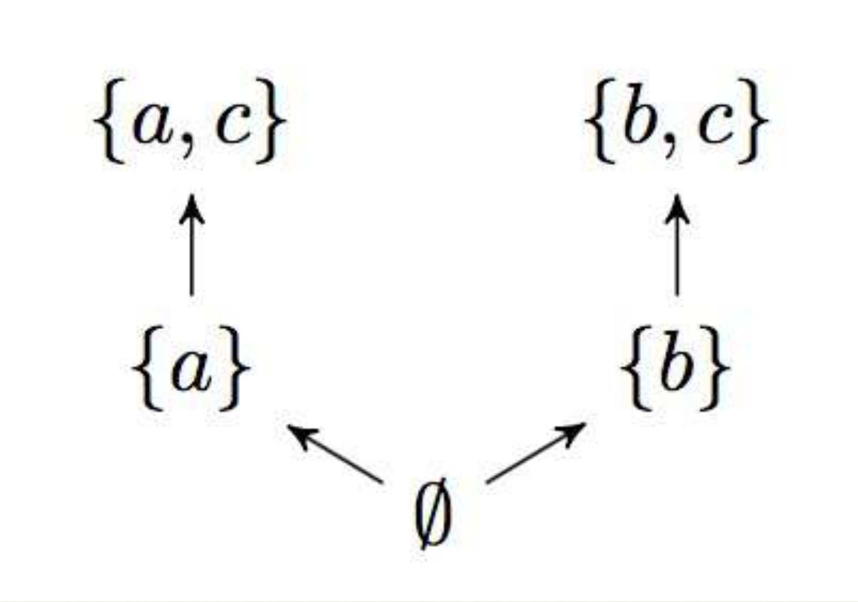
Moving back and forth between connected ESs and weak prime POs induces an isomorphism (actually, an equivalence of categories…)

counter example

Different unconnected ESs may generate the same PO



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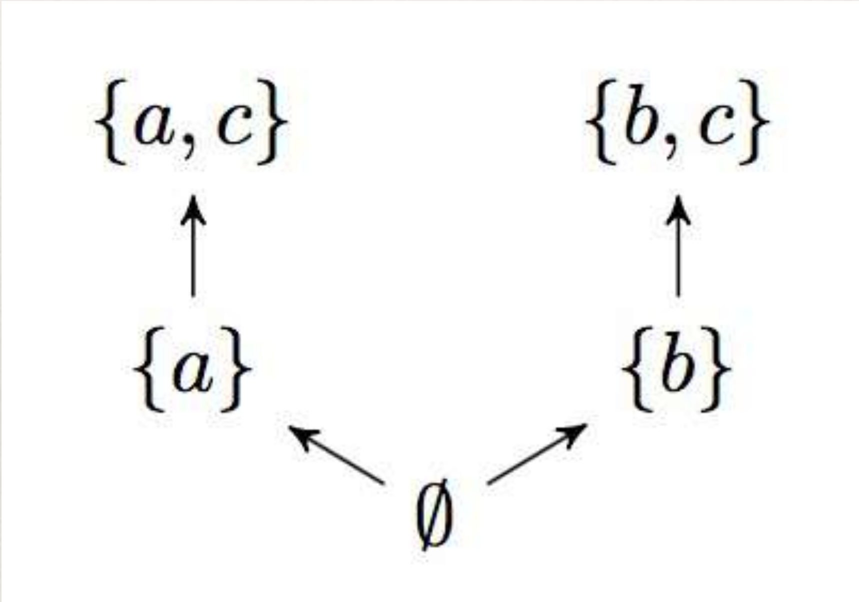

$\emptyset \vdash_0 \{a, b\}$ $\quad a \# b$

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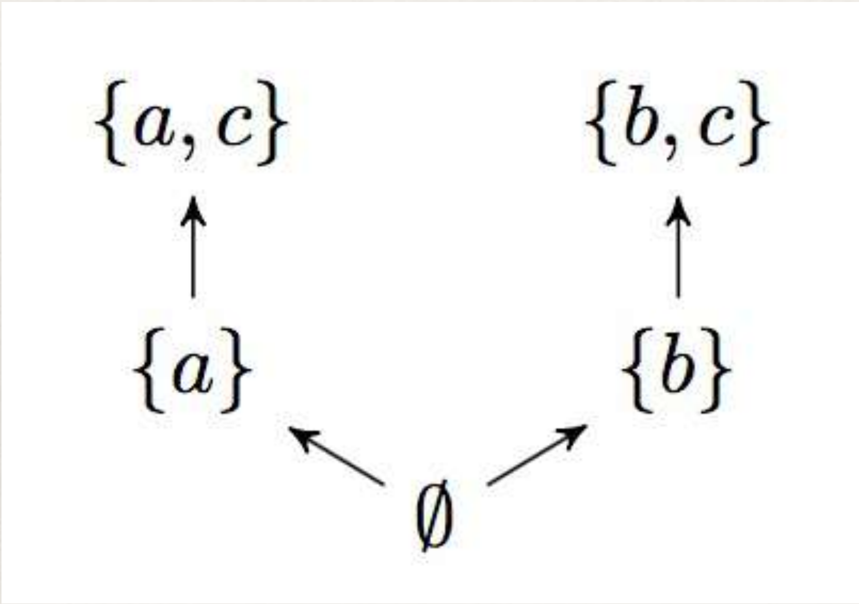

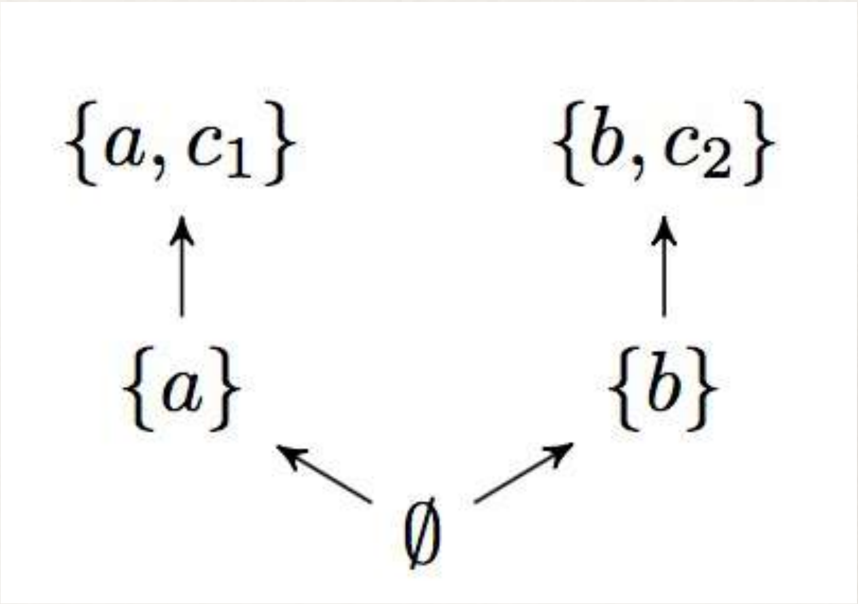

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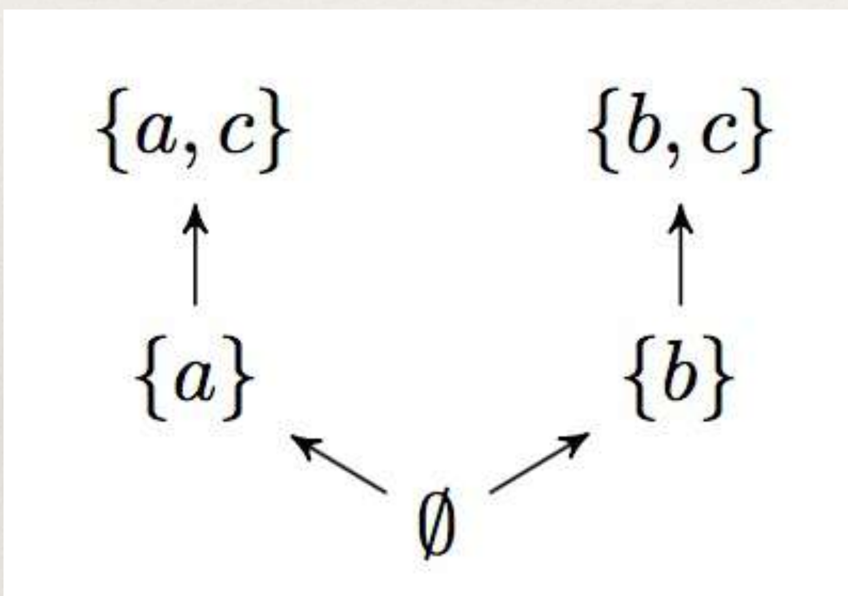

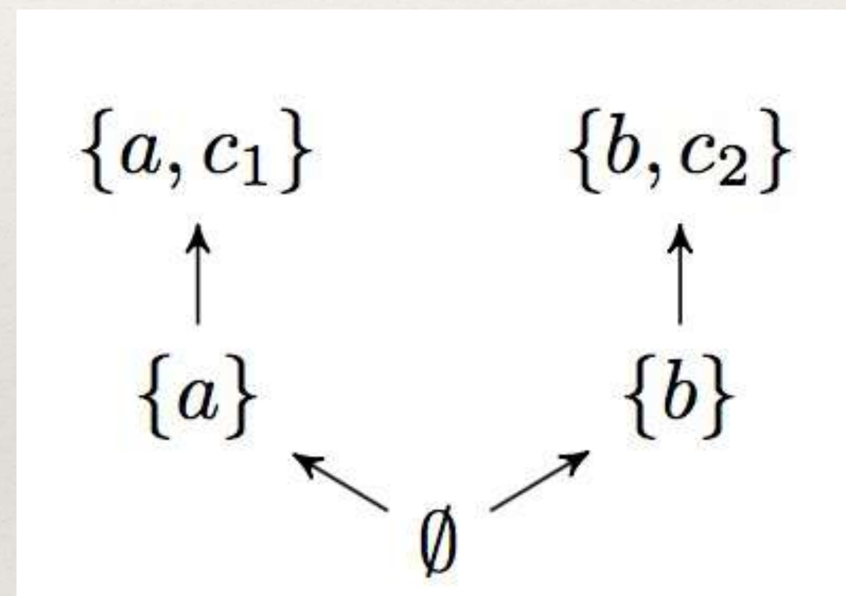

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$\{a\} \vdash_0 c_1$ $\quad \{b\} \vdash_0 c_2$

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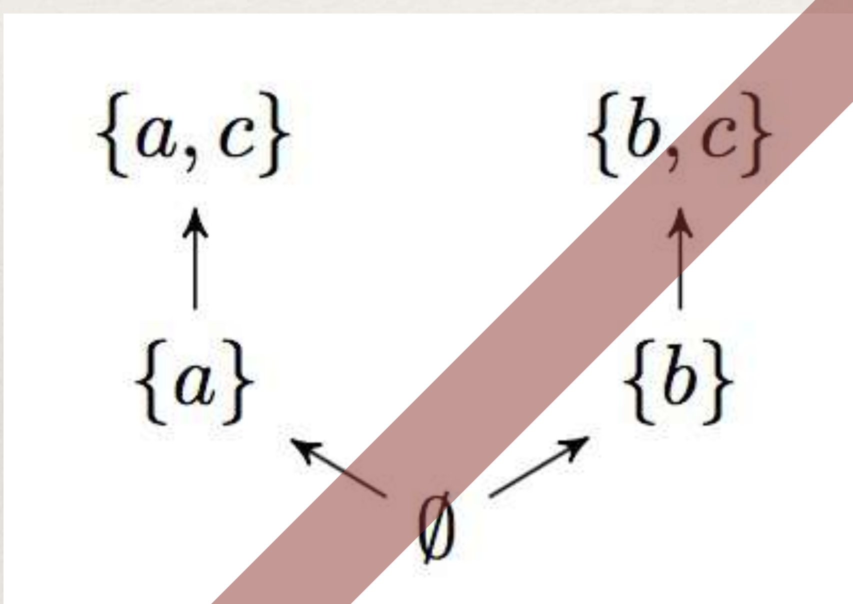

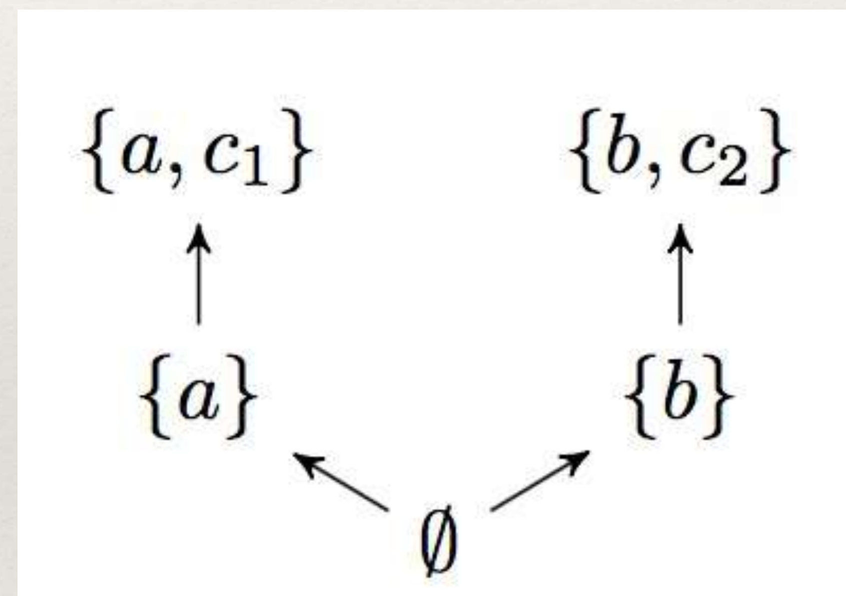

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noteworthy

Connected ESs model those graph rewriting systems that are used in the encoding of process calculi

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Each connected ES can be obtained as the semantics of a graph rewriting system

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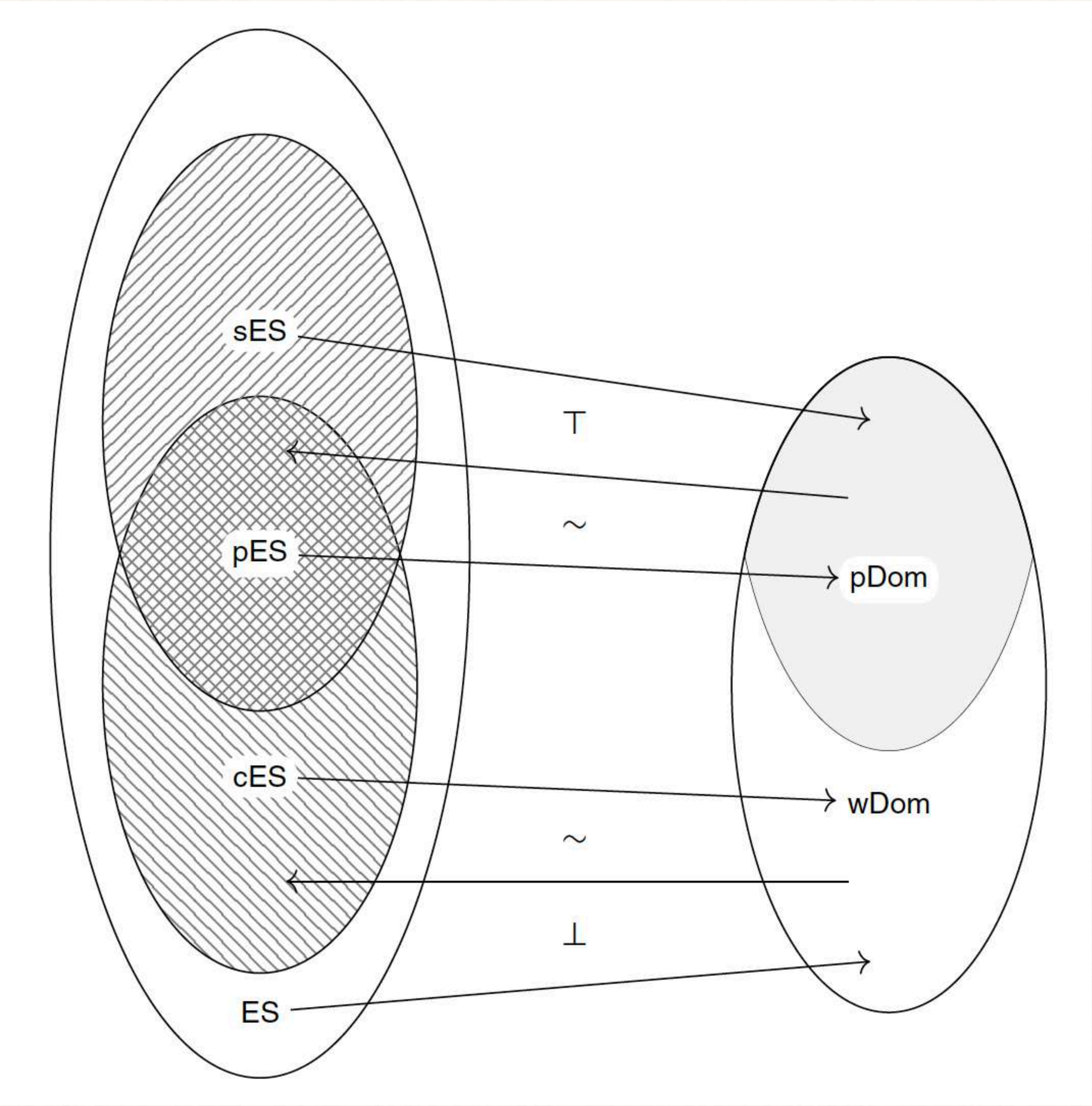
[Claim: connected ESs are exactly the ESs we need!]

[The conflict is not necessarily binary]

some categories, after all…



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$X \vdash e$ and $Y \vdash e$ and

$X \cup Y \cup \{e\}$ consistent

imply $X \cap Y \vdash e$

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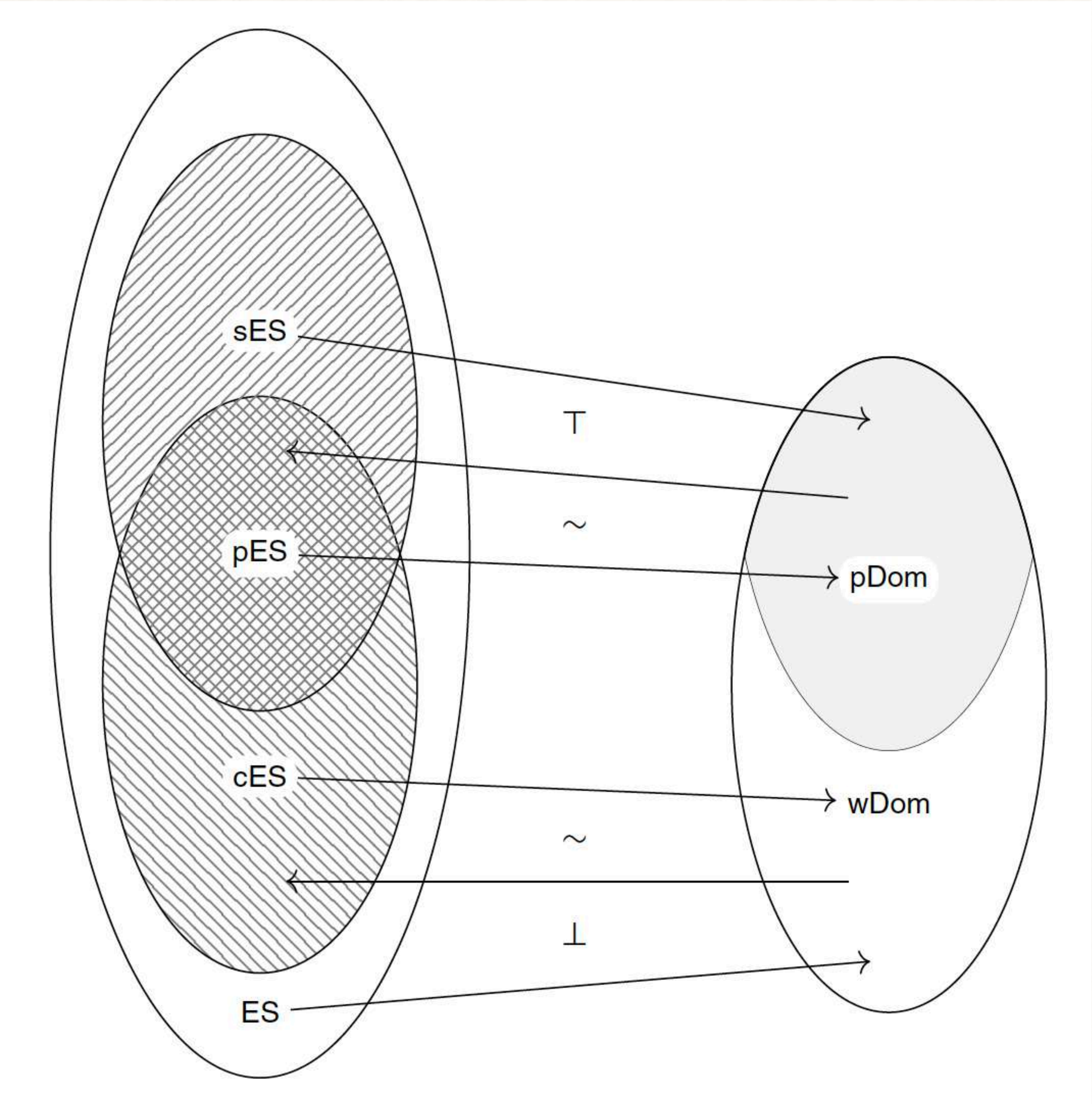

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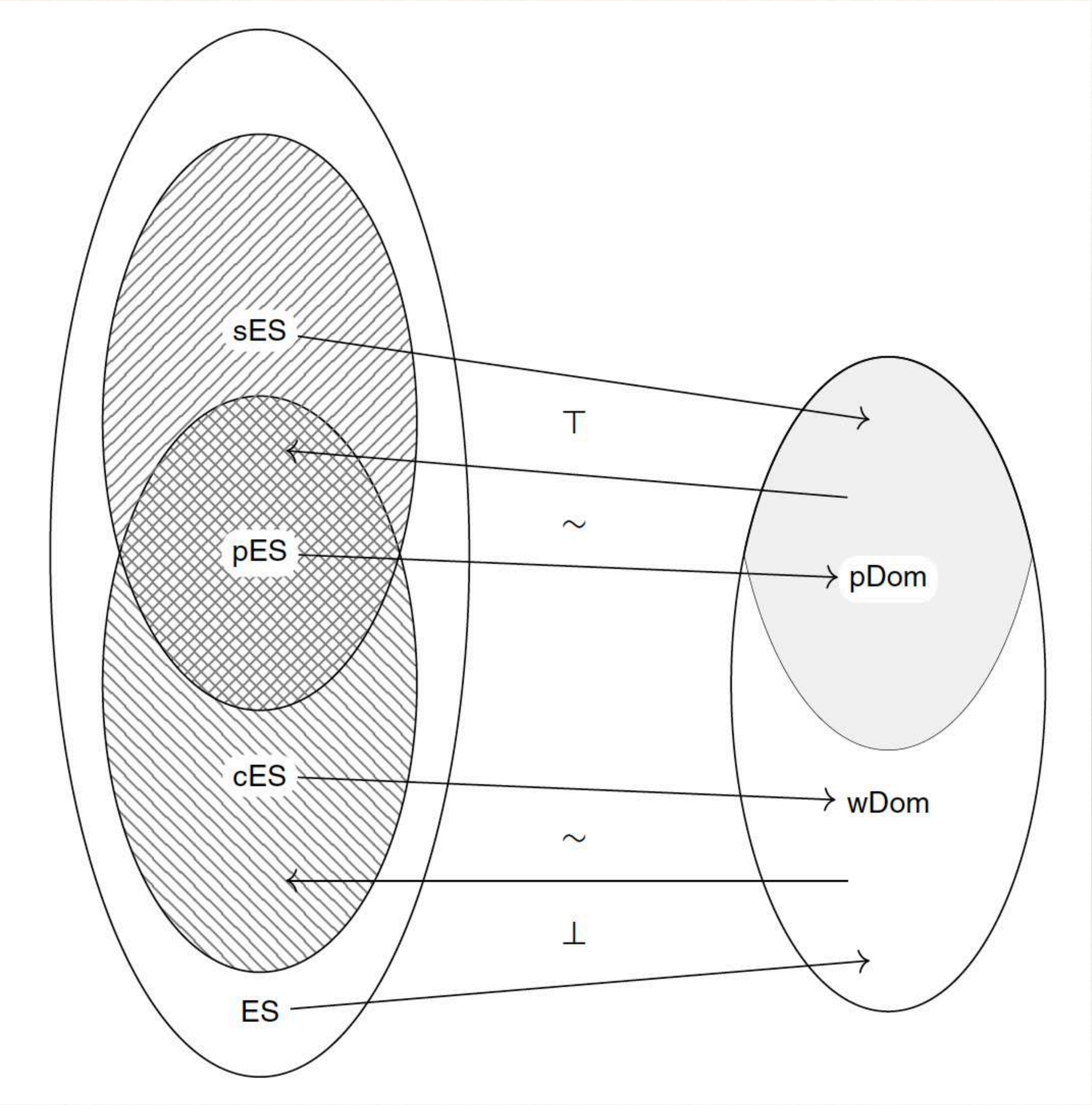

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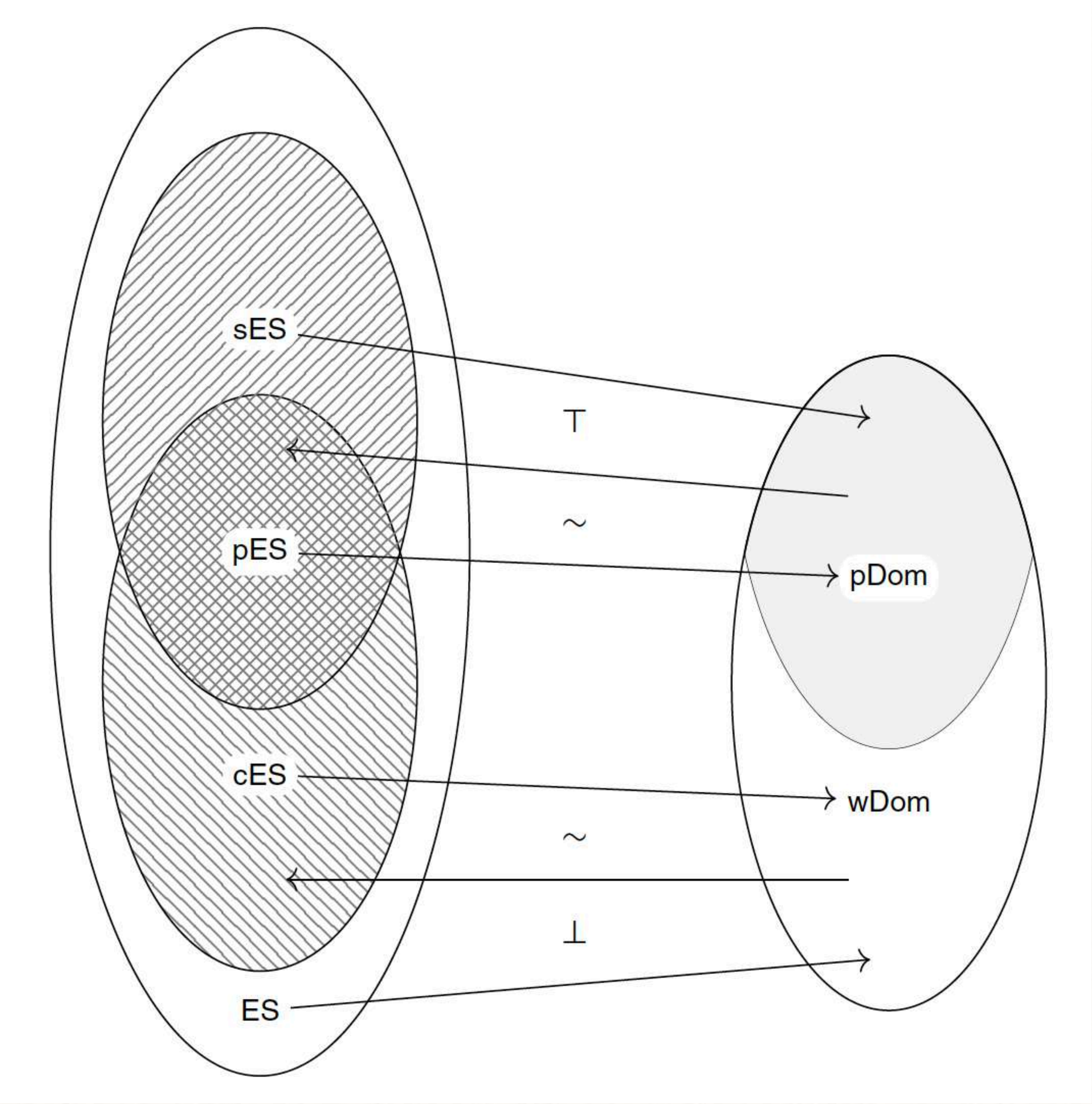

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[stable & unconnected]

Thanks for listening

Questions are welcome!