# Hyper Partial Order Logic 

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## Motivations

Non Interference
$\Sigma=\Sigma_{\text {low }} \uplus \Sigma_{h}$

$\mathcal{L}\left(\mathcal{O}_{\text {low }}(S)\right) \subseteq \mathcal{L}\left(\mathcal{O}_{\text {low }}\left(S \backslash \Sigma_{h}\right)\right)$

$$
\begin{aligned}
& \forall \rho_{1} \cdot h . \rho_{1}^{\prime}, \exists \rho_{2} \in \sigma_{\text {loon }}^{*}, \\
& \mathcal{O}_{\text {low }}\left(\rho_{1} \cdot h . \rho_{1}^{\prime}\right)=\mathcal{O}_{\text {low }}\left(\rho_{2}\right)
\end{aligned}
$$

Pb : cannot be expressed with a LTL, CTL property.


Mantel's framework
Comparison of
language closures (projec-
tions, morphisms,...)
[Mantel00] [D'Souza11, 16]


Hyperproperties
Properties of sets of traces
[Clarkson14, Schneider 10]

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& \forall \rho_{1} \cdot h . \rho_{1}^{\prime}, \exists \rho_{2} \in \sigma_{l o w}^{*}, \\
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Non Interference
$\Sigma=\Sigma_{\text {low }} \uplus \Sigma_{h}$


Mantel's framework
Comparison of Alur's framework: CTL~
language closures (projec- CTL
tions, morphisms,...) + a relation on equivalent
[Mantel00] [D'Souza 11, 16]
events
[Alur07]

$$
\begin{array}{ll} 
& o p_{1}(\mathcal{L}(S)) \subseteq o p_{2}(\mathcal{L}(S)) \\
\wedge & o p_{3}(\mathcal{L}(S)) \subseteq o p_{4}(\mathcal{L}(S)) \\
\wedge & \cdots
\end{array}
$$

Hyperproperties
Properties of sets of traces
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## Local Logics

$\mathcal{O}_{\text {low }}$ projection on events with label in $\{a, b\}$.


## Interleaved setting

Formula of the form $\phi::=X\left(p_{a} \wedge X p_{b}\right)$ does not characterize $O_{1}$

## Partial order setting

Address the shape of causal ordering among events in a single partial order! LD 0

## Local Logics

$\mathcal{O}_{\text {low }}$ projection on events with label in $\{a, b\}$.


$\mathcal{O}\left(O_{1}\right): \quad e_{2} \bullet b$
$\mathcal{O}\left(\mathrm{O}_{2}\right):$
$\mathcal{O}_{\text {low }}$
$f_{1} \bullet a \quad f_{2} \bullet b$

## Interleaved setting :



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Partial order setting

$$
O, e \models \lambda(e)=a \wedge \exists f, e \prec f, \lambda(f)=b
$$

Address the shape of causal ordering among events in a single partial order! LD $_{0}$ [Meenakshi04], TCL $^{-}$[Peled00],...

## Outline

HYPOL : an Hyper Partial Order Logic

## Part 1 : Hypol

- Partial orders, template
- Partial observations
- Hypol : Syntax \& Semantics
- Satisfiability
- Example : causal non-interference


## Part 2 : Model Checking on Petri nets processes

- Unfolding \& processes
- A grammar for unfolding
- Execution graphs
- From Hypol to MSO
- Observable nets


LPO over $\Sigma$
$O=(E, \leq, \lambda)$

- $E$ is a set of events,
- $\leq \subseteq E \times E$ partial order,
- $\lambda: E \rightarrow 2^{\Sigma}$ labeling


## Definition : Isomorphism

$O=(E, \leq, \lambda)$ and
$O^{\prime}=\left(E^{\prime}, \leq^{\prime}, \lambda^{\prime}\right)$ are
isomorphic ( $O \equiv O^{\prime}$ ) iff
$\exists h: E \rightarrow E^{\prime}$ such that
$e \leq e^{\prime} \Longleftrightarrow h(e) \leq^{\prime} h\left(e^{\prime}\right)$
and
$\lambda(e)=\lambda^{\prime}(h(e))$.

LPO over $\Sigma$
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- $E$ is a set of events,
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## Template Matching

$O=(E, \leq, \lambda)$ and
$T=\left(E_{T}, \leq_{T}, \lambda_{T}\right)$
$O$ matches $T$ iff
$\exists h \subseteq E, h: H \rightarrow E_{T}$ such that:

- $\lambda_{T}(h(e)) \subseteq \lambda(e)$,
- $e<_{T} e^{\prime}$ implies $h^{-1}(e)<h^{-1}\left(e^{\prime}\right)$.


## Observation function

mapping $\mathcal{O}: \mathcal{L P O}(\Sigma) \rightarrow \mathcal{L} P O\left(\Sigma^{\prime}\right)$, representing the visible part of the system.


## Observation : examples


$\mathcal{O}_{1}(O)$ : projection on events that carry label $a$ or $b$,
$\mathcal{O}_{2}(O)$ : restriction of $\leq$ to events with an $a$
Main idea : model the observation power of an intruder.

- $A, \Sigma$ atomic propositions
- $\mathcal{T}$ finite set of templates over $A$,
- Obs finite set of observation functions

$$
\begin{aligned}
\phi::= & \operatorname{true}|\neg \phi| \phi_{1} \vee \phi_{2} \\
& \mid \text { match }(\mathcal{O}, T, f) \\
& \left|E X_{D, \mathcal{O}} \phi\right| E X_{\equiv, \mathcal{O}} \phi \\
& \left|\phi_{1} E U_{D, \mathcal{O}} \phi_{2}\right| E G_{D, \mathcal{O}} \phi
\end{aligned}
$$

where $D \subseteq A, T \in \mathcal{T}, f$ is an event of $T$, and $\mathcal{O} \in \mathcal{O} b s$

## Evaluation of formulas

Formulas are evaluated over a set $\mathcal{W}$ of LPOs over $\Sigma$,
$\mathcal{W}$ satisfies $\phi$ iff $\exists O=(E, \leq, \lambda) \in W, e \in \min (O)$,
$O, e \models \phi$

## Satisfiability

A formula $\phi$ is satisfiable iff there exists an universe $\mathcal{W}$ such that $\mathcal{W} \models \phi$
Satisfiability problem : Given $\phi$, is it satisfiable by some universe $\mathcal{W}$ ?

## Model checking

A model $M$ satisfies a formula $\phi$ iff the universe $\mathcal{W}_{M}$ of its executions satisfies $\phi$
Model checking problem : Given $M, \phi$, does $\mathcal{W}_{M} \models \phi$ ?

## Semantics : Matching


$O, e \models \operatorname{match}\left(\mathcal{O}_{1}, T, f\right)$
iff

- one can match $T$ in the observation $\mathcal{O}_{1}(\downarrow e)$ (causal past of $\left.e\right)$.
- with at least a witness mapping $h_{e, f}$ associating $f$ with $e$


## Semantics : $E X_{D, \mathcal{O}}$ and $E U_{D, \mathcal{O}}$

$$
O, e \models \phi E U_{D, \mathcal{O}} \psi
$$

$$
O, e \models E X_{D, \mathcal{O}} \phi
$$



The next observed event satisfies $\phi$


There exists an event in the future that satisfies $\phi$


O

$\mathrm{O}^{\prime}$
$O, e \models E X_{\equiv, \mathcal{O}} \phi$


$\mathrm{O}^{\prime}$

$$
O, e \models E X_{\equiv, \mathcal{O}} \phi
$$


$O, e=E X_{\equiv, \mathcal{O}} \phi$


$O, e \models E X_{\equiv, \mathcal{O}} \phi$

$O, e \models E X_{\equiv, \mathcal{O}} \phi$


O

$\mathrm{O}^{\prime}$

$$
O, e \models E X_{\equiv, \mathcal{O}} \phi
$$

There exists another order $O^{\prime}$ in $\mathcal{W}$ and an event $f$ such that

- $O^{\prime}, f \models \phi$
- $\mathcal{O}$ cannot distinguish the causal past of $e$ and $f$

$$
\mathcal{O}(\uparrow e) \equiv \mathcal{O}(\uparrow e)
$$

## An example : causal Non-Interference

$$
\text { Let } \Sigma=\Sigma_{h i g h} \uplus \Sigma_{l o w} \text { with } \Sigma_{h i g h}=\{h\} \text { and } \Sigma_{l o w}=\{a, b\}
$$

$\mathcal{O}_{\text {low }}$ projection of LPOs on events with label in $\Sigma_{\text {low }}$.


Causal Non-Interference

$$
\begin{aligned}
& T_{\mathrm{h} \leq a}=\bullet^{h} \longrightarrow \bullet^{a} \\
& \operatorname{Pred}_{h}::=\bigvee_{a \in \Sigma} \operatorname{match}\left(\mathcal{O}_{\mathrm{h}, a}, T_{\mathrm{h} \leq a}\right) \\
& \phi_{C N I}::=A G_{\Sigma, i d}\left(\lambda_{\in \Sigma_{\text {high }}} \vee \operatorname{Pred}_{h} \Longrightarrow E X_{\equiv, \mathcal{O}_{\text {low }}}\left(\lambda_{\notin \Sigma_{\text {high }}} \wedge \neg \operatorname{Pred}_{h}\right)\right)
\end{aligned}
$$

If a system satisfies $\phi_{C N}$, then an intruder with observation capacity $\mathcal{O}_{\text {low }}$ cannot differentiate runs with/without $h$.

In particular, a system with behaviors $\mathcal{W}=\left\{O_{1}, O_{2}\right\}$ does not satisfy $\phi_{C N}$ and is not secure

## Theorem

Satisfaibility of Hypol is undecidable

## A PCP encoding :

$I=\left\{\left(x_{1}, y_{1}\right), \ldots,\left(x_{n}, y_{n}\right)\right\}$
$\left(x_{i}, y_{i}\right)$ pair of words in $A^{*}$.
$\exists i_{1} \ldots i_{k}$ such that
$x_{i_{1}} \ldots x_{i_{k}}=y_{i_{1}} \ldots y_{i_{k}}$ ?


PCP instance $I$ has a solution if $\exists O, e$ such that $O, e \models \phi_{I}$

Part 2 : Model Checking on Petri nets Processes

## Definition

A labeled Petri net is a tuple $\mathcal{N}=\left(P, T, F, M_{0}\right)$

- $P$ : set of places,
- $T$ set of transitions
- $F \subseteq P \times T \cup T \times P$ flow relation,
- $M_{0} \in \mathbb{N}^{P}$ is the initial marking.
- $\lambda: T \rightarrow \Sigma$


$W_{1} \in P R(\mathcal{N})$

$W_{1} \in \operatorname{Proc}(\mathcal{N})$

$W_{1} \in \operatorname{Proc}(\mathcal{N})$

$\mathcal{N}$
$W_{1} \in \operatorname{Proc}(\mathcal{N})$

$\mathcal{N}$
$W_{1} \in \operatorname{Proc}(\mathcal{N})$
$O_{1}=\operatorname{Ord}\left(W_{1}\right)$


## Model checking Hypol on Petri nets

Let $\mathcal{N}$ be a Petri net, $P R(\mathcal{N})$ its set of processes
Let $\phi$ be an hypol formula :
$\mathcal{N} \models \phi$ iff $\exists O \in \operatorname{Ord}(P R(\mathcal{N})), e \in \min (O)$ such that $O, e \models \phi$
Unfortunately ....

## Undecidability

Model checking of Hypol properties for (safe) Petri nets is undecidable.

Why : PCP encoding. For every instance $I$ of PCP, can build a net $\mathcal{N}_{I}$ such that $\mathcal{N}_{I} \models \phi_{I}$ iff this instance of PCP has a solution.

## Definition

$\mathcal{N}$ is observable (wrt $\mathcal{O}_{1}, \ldots \mathcal{O}_{k}$ ) iff
i) $\forall \mathcal{O}_{i}$, every cyclic behavior produces something observable by $\mathcal{O}_{i}$ at every iteration,
ii) $\forall \mathcal{O}_{i}$, choices eventually appear in observation after $k_{c}$ steps,
iii) $\forall \mathcal{O}_{i}$, there exists a bound on the size of parallel threads which have identical observation

## Process Semantics : model checking


$\mathcal{O}_{1}$ : projection on events with labels in $\{b, d, e\}$.
i) : $\mathcal{N}$ does not remain unobservable forever

## Process Semantics : model checking


$\mathcal{O}_{1}$ : projection on events with label $a$.
$i)+i i)+i i i) \Longrightarrow$ an event $e$ is always equivalent to a bounded number of events in a bounded past / parallel part

## Theorem

Hypol model-checking is decidable for observable nets.

- show that processes of a nets can be seen as a regular graph $\mathcal{G}_{\mathcal{N}}^{\omega}$
- isomorphism up to observation $\mathcal{O}_{i}$ can be encoded as an additional relation $\xrightarrow{i}$ : gives a new (non regular) graph $G_{\mathcal{N}}$
- Show that Hypol properties of safe nets can be encoded as MSO properties
- Identify a class of $K$-layered nets where $G_{\mathcal{N}}$ is regular Hypol decidable on this class!
- Show that observable nets belong to this class


## Branching processes

Branching processes "unfold" Petri nets

## Definition (Branching <br> Process)

A branching process of $\mathcal{N}=\left(P, T, F, M_{0}, \lambda\right)$ is a triple $B R=\left(O N, \mu, \lambda^{\prime}\right)$

- $O N=\left(B, E, \hat{F}, C u t_{0}\right)$ is an occurrence net,
- $\mu$ is a homomorphism and $\forall e \in E, \lambda^{\prime}(e)=\lambda(\mu(e))$.


The unfolding of $\mathcal{N}, \mathcal{U}(\mathcal{N})$,

is the maximal branching process.
$\mathcal{U}(\mathcal{N})$ can be seen as the union of all processes of $\mathcal{N}$

Unfolding \& processes


Unfolding \& processes










## A grammar for unfolding

Idea of the construction:
Stop unfolding when reaching a marking already drawn


Close to Complete Finite Prefixes
[McMillan95, Esparza02]

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## Graph Grammars

## Theorem

One can effectively build a graph grammar $\mathcal{G}_{\mathcal{N}}$ that generates $\mathcal{U}_{\mathcal{N}}$


## Interesting...

Graphs generated by graph grammars have bounded treewidth

- size of the largest vertex set in a tree decomposition of the graph
- nb. colors needed to generate a graph with a simple graph algebra MSO is decidable for graph grammars

Idea : translate $\phi$ to an equivalent MSO formula ... .... but ... Isomorphism cannot be expressed in MSO.

## Execution Graph

An unfolding, plus explicit representation of isomorphisms

$$
G_{\mathcal{U}_{\mathcal{N}}}=\mathcal{U}_{\mathcal{N}} \uplus\left\{e \xrightarrow{i} f \mid \mathcal{O}_{i}(\downarrow e) \equiv \mathcal{O}_{i}(\downarrow f)\right\}
$$



## Proposition

There exist labeled safe Petri nets and observation functions whose execution graphs are not of bounded treewidth

## Theorem

For every Hypol formula $\phi$ and every safe Petri net $\mathcal{N}$, there exists an MSO formula $\psi$ such that $\mathcal{N} \models \phi$ iff $G_{\mathcal{U}_{\mathcal{N}}} \models \psi$

## Proof idea:

- $e<f, e \leq f, x \leq_{\mathcal{O}} y$ expressible as an MSO property of $G_{\mathcal{U}_{\mathcal{N}}}$.
- $\mathcal{O}_{i}(\downarrow e) \equiv \mathcal{O}_{i}(\downarrow f)$ is a simple relation $e \xrightarrow{i} f$

Then inductive construction.

## Example : $\phi=E X_{D, \mathcal{O}} \phi^{\prime}$

Let $x$ be a variable representing an event
$C$ be a set of variable names already in use
$\operatorname{MSO}(\phi, x, C)=\exists y, x \leq_{\mathcal{O}} y \wedge M S O\left(\phi, y, C^{\prime}\right)$
with

- $y$ is a fresh variable name (w.r.t. $C$ and to the set $C_{x \leq \mathcal{O}}$ of variables used to encode $x \leq_{\mathcal{O}} y$ )
- $C^{\prime}=C \cup\{y\} \cup C_{x \leq \mathcal{O} y}$;


## From Hypol to MSO

Immediate corollaries :
Corollary
Hypol $\backslash E X_{\equiv, \mathcal{O}_{i}}$ is decidable for safe Petri nets
Proof: Equivalence edges are not used. MSO decidable for graph grammars [Courcelle90]
Checking $\mathcal{N} \models \phi=$ checking $\mathcal{G}_{\mathcal{N}} \models \operatorname{MSO}(\phi)$

## Corollary

MSO is undecidable on execution graphs of safe Petri nets
Proof : Consistent with former theorems. Further $G_{\mathcal{U}_{\mathcal{N}}}$ may contain infinite grids minors (a condition for undecidability of MSO [Robertson\&Seymour91])

## Observable nets \& Layeredness

## Distance between events

$\operatorname{dist}(e, f)=$ maximal number of edges between $\{e, f\}$ and their common past

## Balls

The $K-$ Ball of $e$ in $\mathcal{U}_{\mathcal{N}}$ is the set

$$
\operatorname{Ball}_{K}(e)=\{f \in E \mid \operatorname{dist}(e, f) \leq K\}
$$



## Equivalence decision on $K$-layered graphs

## Definition : K-layeredness

$\mathcal{N}$ is $K$-layered for observations
$\mathcal{O}_{1}, \ldots, \mathcal{O}_{q}$ iff :

- the $K$-ball of every event $e$ of $\mathcal{U}_{\mathcal{N}}$ is finite
- $\forall \mathcal{O}_{i}, \operatorname{dist}(e, f)>K$ implies $e \not \equiv_{i} f$
- $e \equiv_{i} f$ can be decided from the contents of $\operatorname{Ball}_{K}(e)$ and Ball $_{K}(f)$



## Proposition

Let $\mathcal{N}$ be a $K$-layered safe Petri net (w.r.t. $\mathcal{O}_{1}, \ldots, \mathcal{O}_{g}$ ). Then, one can effectively compute a graph grammar $\mathcal{G}_{K, \mathcal{N}}$ that recognizes $G_{\mathcal{U}_{\mathcal{N}}}$

Proof idea: Hyperedges memorize $K$-balls of maximal events.

## Equivalence decision on K-layered graphs

Corollary
Hypol model checking is decidable for $K$-layered nets.
Open question
Is $K$-layeredness decidable?
Theorem
Observable nets are $K$-layered for some $K \leq \max \left(2 \cdot k_{c}, 3 \cdot|T|\right)$

## Corollary

Hypol model checking is decidable for Observable nets

## Conclusion

## Contributions

- A new partial order hyperlogic : Hypol
- Hypol Model checking decidable for $K$-Layered nets.
- A decidable subclass : Observable nets


## Open questions

- Complexity?
- Decidability of $K$-layeredness?
- Unbounded Petri nets?
- Other types of regular models?

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