GPS-Posets and the Forbidden Five

Internship Proposal

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Topics of the proposal: Order theory, graph theory, combinatorics

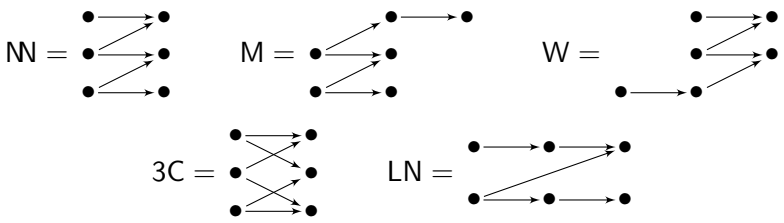
Posets are very simple combinatorial structures: sets equipped with a relation which is reflexive, transitive, and antisymmetric. Despite this simplicity, the theory of posets is rather complicated and many problems remain open.

Together with colleagues in Sheffield, Oslo and Warsaw, the second author of this proposal has recently introduced a new class of *gluing-parallel-symmetric* posets. These are posets generated from points using a parallel composition $\|$ and a new gluing composition $*$. For example,

$$(\bullet\|\bullet) * (\bullet\|\bullet) = \left[\begin{smallmatrix}\bullet\\\bullet\end{smallmatrix}\right] * \left[\begin{smallmatrix}\bullet\\\bullet\end{smallmatrix}\right] = \left[\begin{smallmatrix}\bullet & \bullet\\ \bullet & \bullet\end{smallmatrix}\right]$$

We've done some work on these *gps*-posets, and we know some interesting properties, but one conjecture remains open:

Conjecture. A poset is gps iff it does not contain any of the following induced substructures:



The immediate purpose of this project is to prove the above conjecture. For this, the student will need to understand the precise definition of gps-posets and some of their properties; but otherwise the proof should be entirely combinatorial.

Looking beyond that immediate project, we have introduced gps-posets in the setting of *concurrency theory*, which is concerned with modeling and analysing systems with parallelism. Should the student develop an interest in these things, there will be lots of interesting theory to explore regarding po(m)set automata, higher-dimensional automata, languages of interval orders, etc.