M2 Internship: Variants of Higher-Dimensional Automata

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Topics: Higher-dimensional automata; concurrency theory; automata theory

Prerequisites: Automata theory; logic & verification
Knowledge of algebraic topology and/or category theory is a bonus, but not necessary.

Location: LMF, ENS Paris-Saclay, 4 avenue des Sciences, 91190 Gif-sur-Yvette

Context: Higher-dimensional automata (HDAs) are an extension of automata which allows to model concurrent and parallel executions. Like standard automata they consist of states and transitions, but also of higher-dimensional *cells* (squares, cubes, etc.) which model parallel execution of several events. This gives HDAs a somewhat *geometric* flavor.

HDAs have been introduced in [5, 6], but recent work modernises and simplifies the notion [2] and introduces the basis of a proper *higher-dimensional automata theory*: a Kleene theorem [3]; a Myhill-Nerode theorem [4]; a Büchi-Elgot-Trakhtenbrot theorem [1]; etc. Research in HDAs is progressing fast and in France is done primarily at LMF (Paris-Saclay), LRE (EPITA Paris & Rennes), IRIF (Paris Cité), LIX (École polytechnique), and SAMOVAR (Télécom SudParis).

Languages of HDAs need to account for both the sequential and the concurrent nature of computations. Their elements are thus finite *pomsets* or *partial words*. As an example, Figure 1 displays an HDA consisting of two squares, with three events labeled a, c, and d. Here the a-labeled event is executed concurrently to the sequence $c.d$, so that the language of this HDA will contain the pomset

$$\left[\begin{array}{c} a \\ c \longrightarrow d \end{array}\right].$$

Objectives: Recent work has created a need for types of automata which are more general than HDAs:

- *partial* HDAs in which some faces may be missing (in Fig. 1 for example, the bottom c-transition might be missing, meaning that a may occur before or in parallel with c, but not after c);

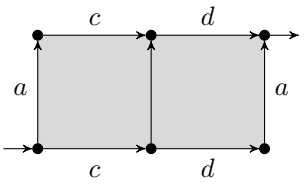

Fig. 1: HDA which executes a in parallel with $c.d$. Initial and accepting cells marked with arrows.

- HDAs with *interfaces* in which faces may be missing, but only in a manner which is consistent with the events (in our example, both c-transitions could be missing, but not just one of them);
- HDAs with *silent transitions* (like for standard automata; but now only some of the events in a higher-dimensional cell may be silent);
- *ST-automata* which do away with the geometric intuition of HDAs altogether (in our example, all faces might be present, but the *link* between the top-left state and the top c-transition might be broken).

The objectives of this internship are

- to clarify these notions and the relationship between them;
- to develop the language theory of these models;
- to develop their applications.

Among these applications are

- HDAs which recognize *infinite* words (ω-HDAs);
- translations between HDA variants and *Petri nets*;
- active *learning* of HDAs and their variants;
- higher-dimensional *timed* automata.

Each of these topics provides rich opportunity for research, according to the student's preferences, and may be subject of a subsequent PhD thesis.

References

[1] Amazigh Amrane, Hugo Bazille, Uli Fahrenberg, and Marie Fortin. Logic and languages of higher-dimensional automata. In Joel D. Day and Florin Manea, editors, *DLT*, volume 14791 of *Lect. Notes Comput. Sci.*, pages 51–67. Springer, 2024.

[2] Uli Fahrenberg, Christian Johansen, Georg Struth, and Krzysztof Ziemiański. Languages of higher-dimensional automata. *Mathematical Structures in Computer Science*, 31(5):575–613, 2021.

[3] Uli Fahrenberg, Christian Johansen, Georg Struth, and Krzysztof Ziemiański. Kleene theorem for higher-dimensional automata. *Logical Methods in Computer Science*, 20(4), 2024.

[4] Uli Fahrenberg and Krzysztof Ziemiański. Myhill-Nerode theorem for higher-dimensional automata. *Fundamenta Informaticae*, 192(3-4):219–259, 2024.

[5] Rob J. van Glabbeek. Bisimulations for higher dimensional automata. Email message, 1991. `http://theory.stanford.edu/~rvg/hda`.

[6] Rob J. van Glabbeek. On the expressiveness of higher dimensional automata. *Theoretical Computer Science*, 356(3):265–290, 2006. See also [7].

[7] Rob J. van Glabbeek. Erratum to "On the expressiveness of higher dimensional automata". *Theoretical Computer Science*, 368(1-2):168–194, 2006.